



MODEL OPTIONS FOR MANDATORY OLD-AGE ANNUITIES

BY

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Corvinus University of Budapest
Individual Ph.D. program
Model options for mandatori old-age annuities
Ph.D. dissertation
dr. Banyár József
Budapest, 2011



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1. FOREWORD

NARROWING THE SUBJECT

The life insurance market of the past few decades in Hungary (but probably in most of the European, or at least in the majority of the Central-Eastern European countries) give the impression at first glance that annuity insurance is a marginal product not worthy of special attention, since people obviously do not tend to purchase such products – disregarding certain specific groups of people like disabled miners.

Though not looking back to the past but forward to the possible future, a completely different view unfolds. With great probability the marginal annuity insurance becomes an important market leading product in Hungary and in general in the Central-European region within a decade, what's more this change has already started and is heavily underway within a specific segment in Hungary– e.g.: in the market of annuity for flat.¹

The reason for the change among many other factors is primarily found in the actual demographic trends. This is why it may happen that a great number of single old-age flat owners (left without heirs, without active-age supporter as a consequence of the demographic situation), wish to convert the flat ownership for annuities, by this giving a kick to the formal business of annuities for flat, and primarily this led to the pension reform at many places in the region (in Hungary in 1997) around the turn of the century. In the course of the reform the mandatory individual pension account was introduced, for this specific private pension institutions for accumulating funds (in Hungary private pension funds) were established, though it was not necessary. These funds must convert the accumulated individual pension funds for annuities after a certain period of time (in Hungary it is 15 years²). Due to this reason the annuity has

¹ The fundamental World Bank publication which set the changing of the Central-Eastern European pension systems (by James /1994/ - editor) deals with annuities among other topics only in its Appendix, though the most recent publications cast an increasing future significance of annuities, respectively that of the annuities market as a result of the abolishment of DC systems; also the so far neglected problems of the annuities phase in contrast with the accumulation phase is worded.

² This is true in spite of that in 2010-2011 the Hungarian private pension system – in response to a governmental initiative – reduced to a fraction of its previous size. But for the remaining system the here explained problems are still valid.



become theoretically and practically an important topic of our day (the term annuity is generally used in this paper as a synonym for life annuity, unless I specify otherwise).

Considering the importance and the complexity of the phenomenon, one can hardly find literature dealing with the subject and this is specifically the case in Hungarian language³ however we can witness a gradual change of the past few years concerning the literature in English language. Therefore exploration of the specific features how annuities operate, the impact of the different factors, the inferences the consequent hazard for service providers, consumers and the regulators as well the exploration of the means of protections against these hazards is important. A separate, important problem is what may be and what is worthy to be regulated is how a coherent regulation may be achieved. Due to the complicated inferences there is a great hazard of ending up with bad, voluntary solutions and this should by all means be avoided.

I do hope that my paper contributes to the above goals, to the development of the of sustainable products for the market and to the drafting of regulation optimally protecting customers, and last but not least promotes the understanding of the specific features of the operation of annuities.

Though I insert my topic into a wider context I do narrow my subject from different aspects. Primarily I deal with life annuities however this is technically often coupled with the certain annuities and the schedule in other words regular withdrawal of money, what is therefore also discussed here. Life annuity is a much broader category in technical sense than what it means as a commonplace term, for example it includes the payment of life annuity premium and certain debt services as well. Here I only deal with life annuities according the commonplace meaning (though at the beginning of the paper I insert those into a broader context of life annuities in technical sense) so I deal with the case when a service provider makes regular

³ After the launching of the private pension funds Réti warned (Réti [1999]) of the incredibly (and needlessly) detailed regulation of the institutional system of the private pension funds, though the regulation of service provided by them is a sketch and he makes a hint that it was not thoroughly developed actuarially either. This was admitted by Stahl too, who actively took part in the elaboration of the regulations (Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002], Stahl [2005]. Erdős (Barabás-Bodor-Erdős-Fehér-Hamecz-Holtzer [2006] says bluntly that “In 1997 at the time of drafting the bill on private pension funds the first service to be provided appeared to be so distant and there was such a hurry with the drafting, that the provisions on the services are hastily thrown together.”



disbursements to a client, and I do not deal with the inverse situation. Life annuities are adequate for orphan's annuity, handicapped annuity, disability annuity, etc., but I narrow down my subject to old-age, that is pension annuities. Among these I only detail the mandatory life annuities, so my supposition is that at the time of retirement the funds on pension accounts must be mandatorily converted into annuity. This criterion is of great importance, because the mandatory conversion into annuity cuts through a vicious circle, what is the reason why voluntary annuity markets are typically insignificant all around the world (compared to the life insurance market, but alone as well). The core of the vicious circle is the extremely strong auto-selection, that is those who do not expect a long life do not buy annuity, rather consume the assets by this leaving the possibility open for eventually leaving heritage behind. Therefore the rentier's lifecycle becomes longer than the average what forces the annuity providers to raise their fees with respect to this, what diverts further people from concluding annuity. Obligation eliminates one of the strongest sources of auto-selection, so this factor shall be calculated with when designing the annuity system. In other words this paper would be completely different with my supposition that converting funds on the individual accounts is voluntary. Nevertheless I further narrow my subject, by supposing that the mandatory annuity system is coupled with the prohibition to differentiate among the insured people. This is also a supposition similarly strong to the obligation but this supposition has a contrary impact: While obligation eliminates the possibility of auto-selection, prohibition of differentiation broadly brings that back, what shall be somehow treated. The options for the providers are totally different with possibilities to differentiate (according to gender, qualification, health condition etc.), or with a legislation in place that prohibits it. Recently the EU policy adopted the prohibition of differentiation, so this shall be dealt with anyhow, and this is the major reason for the mandatory annuity system to fall apart into models. This is presented below.

Above I presented the way I narrowed my thesis. However I tried to indicate the contrary trend too, e.g.: to discuss the subject as generally as it is possible – within the limitations of the above said – I do not intend to narrow the thesis to the Hungarian situation, though most of the examples are naturally from Hungary. My thesis is more- or less generally applicable to such pension systems, where there is mandatory



individual accounts and mandatory annuity system in place. Looking at it from this perspective all the above said is characteristic primarily on the Eastern part of Europe and less on the Western part. The major difference between the parts of Europe is that the part of the pension scheme that operates in the non pay-as-you-go system, what is (more or less) mandatory (or at least covers almost every employee) what is generally referred to as II.nd Pillar, is extremely different. In Western-Europe the employer pension model is wide-spread, where the pension is provided by institutions that are quite close to the employer from organizational and policy point of view (or they form part of the employer), while in Central- Eastern-Europe these are financial providers separate from the employer. In Western-Europe the employer pension are characteristically of DB type (however the DC type is quickly spreading) though in Eastern Europe these providers operate almost exclusively in DC system. In the case of DC systems the rules of annuity provision are not clear - contrary to the DB system – so there is room for analysis, such ones as this paper aims to be.

The aim of the study is to present the possible types of annuity regulatory systems, models that are adequate from technical point of view e.g.: are free from contradiction that serves the interest of clients in the context of widely applied conditions (individual account, mandatory conversion for annuity, prohibitions of differentiation).

The structure of the study is as follows: the first chapter gives an overview of the most important technical questions of annuities, the theoretical basis of annuity calculations, the theoretical relations between certain types of annuities and of other financial products (mainly life insurances). At the end of the chapter I will indicate the sort of theoretical supposition that gives the basis for the annuity calculation and the cases when these are not fulfilled in the practice that is what problems may occur in relation to the cash-flow. The second and the third chapter deals with these problems – what are basically the consequence of prohibition of differentiation – and the possible techniques for the management of these problems. From among the problems I highlight the problems of selection, of choice, and these are detailed in a separate chapter; I try to quantify the negative impact of these problems on the annuity cash-flow. Some of the techniques adequate to treat different problems strengthen each-other, application of other ones exclude each-other, so it is not



possible to build-up the annuity models voluntarily by defining some “lego” bricks, of which arbitrary models can be built up by decision makers. Therefore the fourth chapter gives an overview of different elements that lead to various sorts of consistent annuity model version. I find that all together six consistent annuity models with inherent alternates and decision makers may opt from among them, but different elements of separate models may not be arbitrarily combined. In the last chapter I try to give aspects for making an option from among the models, and make a brief overview of the regulatory problems; I speak about the wider context of annuity models and the regulatory and institutional problems of fund accumulation; and I speak in general pension system framework, within which the systems of individual, capitalized accounts work properly, what I personally prefer.

ANNUITIES IN GENERAL, DEFINITION AND TYPES OF ANNUITY

The term⁴ annuity and life annuity may be used in wider technical, or in narrower, regular sense.

Using the wider sense that is in technical terms annuity is every kind of regular, standardized cash-flow what was preceded by or will be succeeded by some transfer of goods, capital or service as compensation in strict settlement; or it is some voluntary, prolonged time to pay free from compensation. Standardization means that members of the cash-flow (annuity dues) follow one another at regular intervals during non-defined or defined time, and/or until the occurrence of certain defined event(s), and the magnitude is either constant or changes according to a defined pattern. In narrower sense I regard annuity only those regular, standardized cash-flows that are paid not as consideration for a current service, what are paid by a financial institute to one (or more) individuals.

The life annuity is such an annuity, where the start and/or the termination, and the changing of the annuity dues and/or the size of the intervals of the annuity depend

⁴ In this paper annuities are discussed only formally, that is from the point of their structure and target, but it is possible and certainly is very interesting to discuss them according to their history. Regrettably there is very little literature available about is, I have not found any such document in Hungarian language or any document discussing the Hungarian context. The first chapter of Potreba (1997) and of Mackenzie (2206) deals with the early annuities (e.g.: in the antiquity and in modern times). Poterba (1997) primarily focuses on the American history of annuities, what is also included in the publications of the „annuities association” (IRI Annuity Fact Book 2009, IRI (2009).



on the death of annuity members. The term annuity may also be used in technical and regular, that is in narrower sense. In technical sense the annuity is the premium paid for the life insurances, but in narrower sense life insurance is only an annuity the service whereof is the annuity itself.

If I do not signal specifically, the term “annuity” respectively “life annuity” is used in the narrower - that is in the regular - sense in the study, except when I specifically speak about the mathematics of annuities, since the technical sense is self explanatory in that context because at that level there is no difference between in the narrower and the wider sense. As the subject of my study is the life annuity, therefore the term “is used as a synonym of “life annuity”, except at those places where I signal any diversion from this. Since I regard life as insurance, therefore I use the term “insurance” in the sense of life as well.

Annuities – both in wider and in narrower sense – may have indefinitely numerous kind of forms, therefore it is extremely difficult to find such grouping where all possible types are indicated. Consequently I do not strive for completeness but I do try to describe the different types in the possible widest range.⁵

The first and foremost aspect of classifications is whether the disbursement of the does or does not depend on coincidence, what fundamentally means the decease of one or more policy holder. If the answer is yes, we speak about life annuities, if the answer is no, than we speak about certain annuity.

Based on the content certain annuities may be further divided into annuities of defined and non-defined content. The of non-defined content is basically phased withdrawal from the accumulated assets (principle), whose duration is not pre-defined (however the obvious constraint is magnitude of the assets), but there is neither intention for the entire consuming of the capital nor for further accumulation at real value. The annuities of defined duration may be an ever lasting “perpetuity”, or temporary annuities of “financial engineering” (sometimes “banking engineering”). In

⁵ A sort of classification of annuities (different from each other and different (though not fundamentally different) from the one in here can be found in the textbook Life insurance (e.g.: Mehr-Gustavson [1987], Black-Skipper [1994], Banyár [2003b]). From among these the Black-Skipper [1994] classification (and the book itself as well) is widely referred to, this is used for example by Blake as well (Blake [1999]. The recent Encyclopedia of Actuarial Science (Teugles-Sundt (editor) [2004]) similarly to the majority of the literature – rather contains a sort of (not too detailed) annuity listing. See a practical approach to the issue in the issue of Professional Training Service [1997] publication.

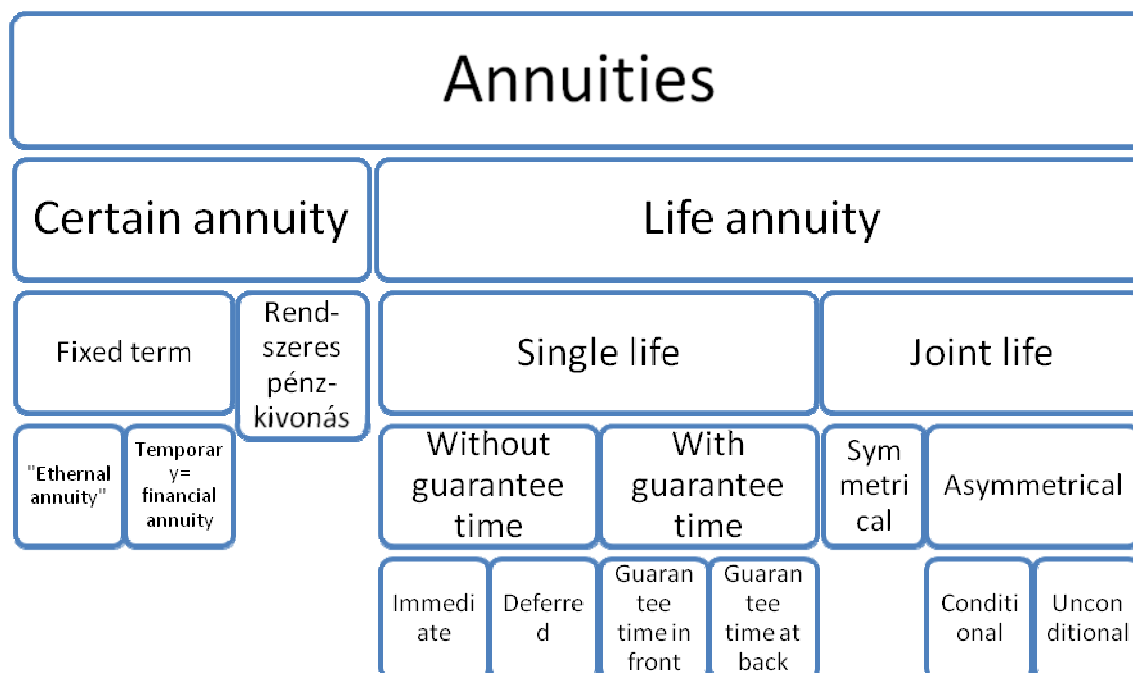


the case of perpetuities the owner of the principal (assets) would only consume, receive as a regular benefit the interest, respectively the yield (or only a portion of that), so the principal is preserved for ever (eventually it even augments). An example – among others – is the land annuity. In the case of financial engineering annuities there is the intention to consume the entire assets during a pre-defined period of time (or to repay the entire debt, respective to accumulate the targeted assets).

Life annuities may be for one or for more lives (that is there may be one or more insured persons). There may be great varieties of annuities of several persons, but generally these may be further classified in groupings according to distinguishing or not distinguishing among the insured persons. The first type may be called asymmetric, the second one symmetric. For the purpose of this paper the most important symmetric, multi-lives insurance is the survivor insurance of two owners, where the disbursement continues until both insured persons (probably spouses) decease. According to a variation the amount decreases following the first death (but not to the half of the amount but to a higher amount). The multi-lives asymmetric annuity insurances may be conditional or unconditional, according to whether starting disbursement of the annuity is linked or not to the decease of the insured person.

The single life annuities start either immediately or deferred and are with front end or back end guarantee time of the term, though naturally most of them are without it.

The figure below includes the most important types – in general and in particular for this study.

**Figure 1: the most important annuity types**

So annuities may be distinguished according to the following aspects (adding to the aforementioned):

- Does it include haphazard elements (certain annuity, life annuity)
- According to the length of term (defined, respectively undefined term, in case of life annuities temporary or life-long respectively)
- Annuities
 - According to the number of insured persons (single, multi-persons)
 - In case of multi-lives annuities according to the related situation of the insured persons (symmetric and asymmetric annuities)
 - In the case of asymmetric annuities does the start of service depend on and insured event or not (conditional or unconditional annuities)
 - Does it contain guaranteed service or not (annuities with or without guarantee time)
 - According to the start of the service (immediately starting or deferred annuities)

We have to note though in relation with Figure 1 that it is incomplete, and within a type it does not contain all the possibilities, that is the above list of aspects can be extended. Though the multi lives annuities characteristically go without guarantee



time (as indicated in the figure), theoretically these may be supplemented with guarantee time. Both multi lives annuities, and annuities with guarantee time may be deferred, though they characteristically start immediately – and a guarantee time at the beginning in the case of deferred annuities is not a particularly handy solution. In principle the symmetric insurances may also be conditional, but it is much more difficult to find examples for those cases. The figure might be continued, and in certain sense I do continue it in the study below, however I fundamentally remain within the above lines.

Nevertheless important and below strongly applied features remain external to the figure:

- Frequency of the annuity. This in principle may be any kind, though in practice there are two, often applied frequencies: monthly and annual. The pension, survivor, orphan, disabled, etc. benefits are almost exclusively monthly payments.⁶
- Is the annuity due at the beginning or the end of the annuity period? Depending on these options one can speak about preliminary and posteriori annuities. This aspect is purely of technical one.
- Though in the case of annuities the default is that the different annuity dues are of the same magnitude, but one can imagine a different situation. In our days it is only natural that annual indexation of the portion of the yield beyond the computed one is credited based on the capital investment that provides the basis for annuity is not considered as a diversion from annuity dues of identical magnitude. However it is possible that there is some sort of trend in the annuities beyond that as well, e.g. the annuity dues increase (eventually decrease) in each year by a pre-defined amount, or in a pre-defined proportion ratio. It is also possible, that this increment is continuous already during the course of the year, or it may happen in longer periods of

⁶ That is naturally aligned with the general schedule of wage and utility bill payments as fundamental determinant. (Though it is to be noted, that it is a relatively new phenomenon, until the middle of the past century the weekly pay was more frequent, later the fortnightly payment was in place for a while, what obviously better served the timing of consumption of smaller salaries than the rarer salary payment. Therefore – especially in the case of lower pensions a more frequent than monthly pension payment may be considered, the argument against may be the higher related cost, respectively the „set” schedule of utility bills.)



time than a year. As for my part only these changes are considered imaginable within the annuity and I do think that the facility is external to the definition of annuity, when a financial service provider pays the service otherwise payable as one sum in three, significantly decreasing installment, so that by the “annuity payment” the policy holder would enjoy a more favorable tax treatment.

- It is also a default, that this one sum (regularly decreasing, increasing) annuity due is denominated in HUF (in general respectively: in unit of money), though internationally (not yet in Hungary) there is the “variable annuity” following the pattern of investment unit based the life insurances, where the service is not defined in HUF, but in units of variable value (constant, or decreasing, eventually increasing number of units).
- In fact no change of the annuity structure is made, still it is important to mention that contrary to the mainstream custom of hundred years, according to which premium is different only according to age and gender, impaired annuities appeared already in the UK, USA with diverse premium depending on the health condition. (As the regulator concept also appeared that certain type of differentiation – e.g.: according to gender – is prohibited).
- It is also important to mention – though I will not reach out to the subject in my paper – the above definition of annuity is extended in certain sense to the annuity received in Long Term Care, namely its start is conditional, and this condition is not the decease of the policy holder, but a certain physical, mental condition (is unable to take adequate care of himself due to his old age). Because of the specific nature of the cash-flow the extension of the traditional definition of annuity is – in certain sense – the (internationally unique) life annuity for flat – its regulation has just started in Hungary – though I will not further deal with it either.
- It is worth mentioning an early, historic form of annuities – though I will not deal with it – that is the “tontine” in certain sense this gave the basis for the contemporary annuities. Its original form was suggested by an Italian medical doctor, Tonti serving in the French royal court in the middle of the



seventeenth century. According to this a few rich men would collectively lend a bigger amount – each man a nice sum – on long term, the interest of which would be annually paid by the treasury. The interest payable to the deceased lender would be distributed among the ones still alive, so they would gradually receive ever increasing interest amounts and finally the last survivor would receive the interest on the total lent amount. When this person dies too, the principle would get into the property of the king (or in a commercial version that would also become the property of the last survivor). So the solution forges the annuity and the game. Tontina quickly spread all over the world, gradually the element of game started to dominate the facility, as a result these became forbidden almost everywhere and they only exist in France where they were originally implemented.

- Finally let me mention an interesting hybrid, the interfertile of the certain annuity and of life annuity, that is the annually recalculated certain annuity. This is such a certain annuity, the term of which is the still existing expected lifecycle of the person who is the annuitant. Since this annually changes, therefore it is annually recalculated and the annuity due is adequately modified (diminished).

Due to constrained length I do not reach out to the mathematics of annuities in this paper (except for the obvious parts dealing with selection). This can be found in technical literature (partly in my own books). The most frequently referred actuarial literature publications in English language, dealing with the mathematics of annuities as well are the English Neill [1989] of 1977 (unchanged reprint of the 1977 issue) and the American Bowers-Gerber-Hickman-Jones-Nesbitt of 1986 [1986]. One of the (Swiss) authors of this latter one summarized very briefly and consistently at a very high level the mathematics of annuities in a frequently referred book (Gerber [1995]). From the more recent literature the Canadian Milevsky's book deals explicitly with annuities (Milevsky [2006]).

In Neil's book the classic explanation of annuity mathematics (with the life annuities) can be found (presumably he might have taken it from the traditions). This sort of explanation is characteristic in the respective Hungarian technical literature as well (e.g.: Krekó [1994], Michaletzky [1997], Banyár [1994] and [2003]. Neil's book is



complemented by the textbooks of Hungarian courses of the beginning of the nineties, what was based on the translated English actuarial training textbooks (McCutcheon [1991], Neill [1991a] and [1991b].)

Two further elaborations are worthy to mention from the classical English life insurance technical literature, the Booth-Chadburn-Cooper-Haberman-James [1999] mathematics aimed to be the new summary of English insurance mathematics and the 2004, huge, 3 volume Actuarial Encyclopedia (Teugels-Sundt (editor) [2004].

For us, Hungarians the Hungarian technical literature has the advantage that we can look back to a much longer period here, than in the case of the one in English language, since the older literature is more available in Hungarian language. Already in 1896 a thick volume of “Political mathematics” appeared in Hungarian language, what attracted great attention (Weninger [1869]), what gives an overview of the mathematics of life insurances – within this of annuities.

The first complete work of the Hungarian insurance mathematics, already using standard marking, what can be regarded as the basic document is the Political mathematics Part 2 by Bein,Károly, Bogyó,Samu and Havas, Miksa – with the title: The theory and practice of life, disability and pension insurance published by the Franklin társulat in Budapest in 1907 (Bein-Bogyó-havas [1907]).

In the more recent Hungarian insurance mathematics literature (beyond my own books – Banyár [1994] and [2003]) Krekó, Béla (Krekó [1994] and Michaletzky, György (Michaletzky [1997] wrote in general about the mathematics of life insurances, the latter one focusing primarily on annuities.

POSSIBLE PROBLEMS OF ANNUITY CASH-FLOW

- The following implicit conditions are (and to a certain extent are necessarily) used when noting down the cash-flow formula of an annuity:
- The actual mortality of the annuitant is identical with the supposed, that is the “projected” mortality applied for the calculation; this can be split up to four “sub-assumptions”:
 - The composition of different sub-groups of annuitants with diverse risks is not different from the one used at the calculation



- In the mortality change there is no trend unforeseen and not calculated with in the mortality panel (especially no greater decrease than the calculated one)
- Beyond the annual variation that follows from the small stock, there is no variation in the mortality compared to the one planned (small stock, what is not big enough in respect of the following item)
- The number of annuitants is high enough so that the round number of deceased, generate exactly the necessary fraction of provisions
- Everybody buys annuity insurance of identical amount, that is the stock of annuitant is homogenous
- The member of the annuity is constant in time, it does not change

Naturally all the above conditions must be lifted in practice resulting problems must be handled either in the formula, or in the practice of calculation, respectively in the method of generating provisions. Hereafter I make an effort to elaborate on problems resulting from the lifting of the above conditions, and on the possible methods of treatment. Above I have enumerated the following problems:

- The issue of differentiation of annuitants (insured)
- The issue of selection of annuitants, treatment of selections effects
- The issue of the applied mortality panel
- The issue of the treatment of longevity, respectively in general of the mortality risk
- The issue of the treatment of the small stock
- The issue of the homogeneity of the annuitant stock
- The issue of indexing the annuity what is strictly related with the issue of technical interest rate present in the annuity formula, respectively the issue of the return on the investment.

Below I analyze these problems with paying special attention to the issue of selection with respect to its significance, then I reach out to other important questions of annuities, e.g.: the problems of regulation.



2. THE TREATMENT OF ANNUITY CASH-FLOW RELATED PROBLEMS

2.1. DIFFERENTIATION OF THE POLICY HOLDERS

2.1.1. Differentiation and homogenization in the case of life insurances

One of the most important methods is, by which the insurer achieves that the calculated premium of the insurance is sufficient to cover the services is, that different premium is set for different segments of the risk community markedly different from each-other; in other words policy holders are differentiated by risk. In the case of life insurances – apart from the annuity insurance – this traditionally is carried out so, that two tables of premium are determined for each product, one for men and one for women, within this different premium is set by the age of entry - accordingly the default is the differentiation by age and gender.⁷ Beyond this risk assessment is generally (but not always) carried out, where efforts are made for the quantification of risks resulting from individual, health condition, luxury custom (drinking, smoking, consumption of drugs, etc.), occupation (e.g.: dangerous vocation – miner, policeman, etc.), and sports, hobby (e.g.: extreme sports), and if it is necessary the premium, defined on the basis of the calculation made for age and gender, is raised, in other words the policy holders are further differentiated by risk.

The essence of differentiation may also be expressed so, that by its help the risk community inhomogeneous from the point of risk is split into homogeneous (or at least more homogeneous) parts at least compared to the entire risk community. Consequently differentiation means at the same time homogenization (at least on the level of groups of risk communities).

Nevertheless differentiation can not always be carried out – due to different reasons, or it is not always possible to create adequately high number of risk communities with sufficient number of members. In such cases homogenization of the risk community may be an independent solution, what is the opposite of differentiation in certain aspect, though from other point of view these assume or

⁷ Naturally the premium depends naturally (typically on a linear way) on the the insurance amount, the content of the insurance (if there may be any difference in this respect at all – e.g.: in the case of life-long annuities we may take that they have only one type of content) but difference is not due to the different risk of the policy holders.



complement each-other). In the course of homogenization risks deemed to be unmanageable, or individuals themselves that are deemed to be of unmanageable risk are (temporarily or finally) excluded from the insurance, from the risk community respectively (waiting time, exclusion, acquaintance, refusal respectively), eventually lower and upper limits are defined for the possible amount of insurance, that is the extremes of the risks are tried to be cut off. (I will also detail this type of method in relation to the annuities.)

Though differentiation and homogenization are generally parallel applied, but these can replace each other to a certain extent. If there is less differentiation (e.g.: due to business or cost considerations, to simplify underwriting an insurance policy, the importance of homogenization increases, , though by applying differentiation in wider sense, risk otherwise excluded, or policy holders of extreme risk can also be insured by the insurance company.

2.1.2. Differentiation in the case of voluntary annuity insurance

The significant difference between the above cases and in the case of voluntary annuity insurance that traditionally no risk assessment is applied (neither exclusion, respectively refusal). That is different premium is applied based on the age and gender (possibly lower and/or upper limit is set for the possible insurance amount – see below), but the health condition, occupation, luxury custom etc. are not assessed. Instead the basic supposition is that annuity insurance is by definition made by those having the best life expectancy that is whose expected remaining life would be longer than the average. Therefore mortality is not calculated from the public mortality table but from the annuitant select mortality table that indicates significantly lower mortality.

The reason for the above practice is that the risk of annuity insurance is precisely the contrary to all the other, traditional life insurances (death, mixed, term fix, whole life); that is the risk to be managed is not that the insured person dies much sooner than the average, but the contrary, what otherwise is a completely normal human ambition. Therefore the traditional risk assessment methods and the related aspect can not be applied in this case, since in the case of annuity insurances the otherwise



dangerous client is the best client. (Therefore no exclusion or refusal of client is applied.)

The consequence of the above practice is, that conclusion of annuity insurance is mainly worth for those, who have reasonable cause to expect long life, consequently on the one hand this practice strictly constraints the potential range of clients, and on the other hand it justifies the insurer's preliminary assumption of the client's long life. Naturally this does not apply in the case of mandatory annuity insurances (and the mandatory annuity is one example), since people have no possibility to choose the issue of concluding.

Recently some of the insurers became irritated by the consequence of the traditional practice since that constrained the basis of clients, so the so called impaired annuity, with premium differentiated by the health condition of the clients showed up in the Anglo-Saxon markets. The logic behind is, that clients suffering from certain, proven sickness, damage of health are offered preferential tariff, that is the health status (beyond age and gender) is taken into consideration in the premium. Naturally this already presupposes certain risk assessment, the logic of which is precisely the opposite of the traditional one: those simulating illness/disability must be selected and not the ones indicating themselves healthy. The phenomenon may also be interpreted so, that similarly to other sectors of the voluntary insurance, in the field of annuities the market competition led towards the differentiation of the insured people.⁸

We have to recon that while the traditional risk-assessment penalizes attitudes, custom generally regarded as negative (unhealthy conduct of life, harmful passion), in the case of impaired annuities these proven negative habits may provide basis for preferential premium.⁹

⁸ Slightly ahead of the future, the question may be raised whether in the case of voluntary annuity insurances why differentiation is made according to the health status and not according to educational attainment what is precisely contrary to what is sharply raised in respect to the mandatory annuities in the latter part of the study. Probably differentiation according to educational attainment does not take place in the case of free market annuities because annuity insurance is voluntarily concluded almost exclusively by those having diploma of higher education, while in the mandatory system this is obligatory for those of lower education as well.

⁹ Though no impaired annuity is necessarily needed for this, as indicated by the braking announcement in the Hungarian boulevard media. According to this "the Paerel Leven Dutch voluntary supplementary pension insurance fund envisaged a higher benefit for addict smokers with respect to the fact that their life expectancy is shorter." See: <http://www.stop.hu/articles/article.php?id=284988> , March 1, 2008 Source: MTI



In summary differentiation in the case of annuities may be envisaged according to those important factors, which have a high impact on the remaining life expectancy - so on the premium of the annuity. These are:

- Residence, housing
- Workplace, nature of work (e.g.: physical, intellectual, etc. work)
- Educational attainment
- Health status (e.g.: disability, sickness)
- Life style, luxury custom, sports
- Marital status
- Income (via this the magnitude of the expected annuity)
- Self selection – provided the insured person has a choice

It is important to note, that these factors are not independent from each other; for example the health status is in relation with the life style, probably with housing conditions and educational attainment and vice versa, etc. This must be taken into consideration at the final determination of factors for differentiations. In the following I examine rank the possible use of these factors (with the exception of self selection, what I detail in the coming chapter).

2.1.3. Options for differentiation of mandatory annuities

Based on the above said, the pure actuarial approach would require a deeper differentiation in groups of diverse risk, since the calculation becomes ever more stable by deepening the differentiation. In addition to that the stochastic equivalence between the service received and the premium paid becomes valid for ever narrower groups, in other words by the deepening of the differentiation the annuity may be called increasingly correct. On the other hand reference made to the stability of the calculation naturally means as well that the less differentiated the annuity the calculation is less stable – at least if several, competing services are considered, and some mechanism is all the more needed to manage this instability.

At the moment nobody doubts that annuity service provided for identical assets must be differentiated according to age. This is really fortunate from regulatory point of view, because in the lack of this differentiation would either make a mandatory annuity system unmanageable, or such drastic measures would be required (basically



would change the absolutely uniform age of retirement), from which the Hungarian theory and practice of pension system have already gone far away. As regards any other differentiation the different opinions are totally far from each other.

Looking at this issue from the angle of possibilities for differentiation as regards annuities the major obstacles of differentiation (apart from by age) are not independent from each other, there is some overlapping among them and therefore these (theoretically) might be listed differently. Primarily due to the overlapping I do not highlight the general problem of differentiation by age as a special obstacle, though the below list covers this issue.

1. Connection to the annuity of Pillar I
2. EU regulation
3. Equity problems – relation to other systems
4. Incentive for non desired trend
5. Available data, assess ability of the given parameter

Let us analyze them in this order.

1. Private pension funds were established with the aim that the annuity purchased from the assets accumulated in them, complements the annuity received from Pillar I, where this latter one is the bigger item within the pension. Though it was not spelled out that the annuity of the Pillar II must have the same principle as Pillar I, (what's more, certain factors, as the possibility to opt from different kinds of annuities, suggested just the contrary), it is a fairly logical requirement, that the two parts of an individual's pension matches each other, that is the major parameters, like differentiation must be in harmony in the two pillars. Looking at the annuity of Pillar I from the angle of differentiation, we may say, that there is a differentiation by age in it – though not in a perfectly correct way actuarially, but it exists, however differentiation by age is about to fade¹⁰ away - practically we may assume that it does not exist. There exists the differentiation by the rate of accumulated (income) eligibility that is the digressive range but this is also fading away. There is no differentiation by



residence, educational attainment, health status, marital status, life style, luxury custom but to a certain extent there exists differentiation by the nature of work so that in certain vocations (miners, dance artists, members of armed forces) preferential retirement age is set. Though this preferential age is not calculated on the ground lifetime different from the average, but to a certain extent on voluntary basis, or other factors (e.g.: in the case of dance artists the possibility to conduct the given vocation¹¹).

So it may be assumed that (in the sense of wording with regard to the characteristics of Pillar II) in Pillar I there is no (neither foreseen) differentiation by gender and by the magnitude of accumulated assets, neither any other differentiation exists except for differentiation by workplace, respectively the nature of the work, but differentiation does not happen in an actuarially correct manner.

So Pillar I claims for its part, against the annuity of Pillar II, that beyond differentiation by age and by certain groups of occupation, no other aspect shall be included. This issue is specifically sharply raised when it comes to differentiation by gender, as beyond the age annuities are differentiated according to this criterion by the Hungarian insurers, and there are strong arguments in favor of continuing this practice. Probably this is the reason why the act on private pension funds always provided that the mandatory annuity shall be calculated with a unisex mortality table (so the general practice of the private insurance shall not be applied in mandatory annuities).

Though the claim raised by Pillar I is clear, still it shall not necessarily restrict options for differentiation in Pillar II. Nonetheless the use of unisex mortality table is such a strong claim, from which probably no divergence is possible.

¹⁰ Fading away, because the lower retirement age of women may be considered as a type of differentiation by gender even though this moved annuity further away from the actuarially correct one.

¹¹ Combined with the voluntary assumption that the person will not be able to conduct any other occupation to earn his living, consequently he must retire.



2. The EU regulation characteristically does not respect the options for differentiation allowed or prohibited for insurance products (so for annuities) by member states; there is one important exemption though, the differentiation by gender. The so called Gender Directive adopted in 2004 - COUNCIL DIRECTIVE 2004/113/EC of 13 December 2004 implementing the principle of equal treatment between men and women in the access to and supply of goods and services – prohibits the application of different premium calculation according to gender – though not with immediate effect, and with regard to the specific regulatory preliminaries of the member states. The justification of the provision according to item (18) and (19) of the Preamble justifies it as follows:

“(18) The use of actuarial factors related to gender is widespread in the provision of insurance and other related financial services. In order to ensure equal treatment between men and women, the use of gender as an actuarial factor should not result in differences in individuals' premiums and benefits. To avoid a sudden readjustment of the market, the implementation of this rule should apply only to new contracts concluded after the date of transposition of this Directive.

(19) Certain categories of risks may vary between the genders. In some cases, gender is one but not necessarily the only determining factor in the assessment of risks insured. For contracts ensuring those types of risks, Member States may decide to permit exemptions from the rule of unisex premiums and benefits, as long as they can ensure that underlying actuarial and statistical data, on which the calculations are based, are reliable, regularly up-dated and available to the public. Exemptions are allowed only where national legislation has not already applied the unisex rule. Five years after transposition of this Directive, Member States should re-examine the justification for these exemptions, taking into account the most recent actuarial and statistical data and a report by the Commission three years after the date of transposition of this Directive. In article 5 of the Directive it is precisely provided:

Actuarial factors



(1) Member States shall ensure that in all new contracts concluded after 21 December 2007 at the latest, the use of gender as a factor in the calculation of premiums and benefits for the purposes of insurance and related financial services shall not result in differences in individuals' premiums and benefits.

(2) Notwithstanding paragraph 1, Member States (as Hungary) may decide before 21 December 2007 to permit proportionate differences in individuals' premiums and benefits where the use of gender is a determining factor in the assessment of risk based on relevant and accurate actuarial and statistical data. The Member States concerned shall inform the Commission and ensure that accurate data relevant to the use of gender as a determining actuarial factor are compiled, published and regularly updated. These Member States shall review their decision five years after 21 December 2007, taking into account the Commission report referred to in Article 16, and shall forward the results of this review to the Commission.

So in summary: however member states must prohibit discrimination by gender in the premium calculation, but they are free to decide temporarily not to implement it in case of certain types of insurances. This temporary exemption may theoretically endlessly prolonged, but should they not do that, this exemption terminates and the unisex tables must be applied.

This was the situation until 1st of March 2011, when the Court of Justice of the European Union made this article invalid with effect from 21 December 2012. It means, that in the future the differentiation according to gender is not an option in case of insurance products. (Court of Justice of the European Union [2011]) However in theory this rule is not valid for the mandatory annuity because it belongs to another directive than the insurance directives, but it is valid for the mandatory annuities provided by insurance companies. Although it seems that independently from the gender directive – especially because of the reasons explained by the previous chapter – in case of mandatory annuity it has to apply an unisex rate.



3. Though some of the above listed aspects of discrimination may be justified and correct in itself, mainly taking into consideration life style, luxury habits, sports and in relation to these health status, but may lead to inequity with having regard to the existing social insurance systems in place. In the case of annuity if somebody does not have harmful customs and has a healthy lifestyle, makes sports, expectedly would have a longer lifetime that means he may buy the same annuity only with higher contribution. This is correct if this same factor is taken into account at other systems as well – with opposite direction – because most probably a healthy person would acquire less health care service, consequently for him it would be justified to decrease his health care contribution, and it should be increased for those having an unhealthy life style¹². So taking health status into account is equitable if it is taken into account in the health care system too.
4. In strict relation to the above health status considerations may show up in the annuity premium for the individual, that the annuity provider penalizes the healthy life style and rewards the unhealthy one, the harmful custom (smoking, alcohol, drugs). This in itself may be an incentive for the development of such custom, or at least may give a good argument for not changing the existing bad habits. This non-desired incentive may have a milder, but equally dangerous impact from annuity point of view, that it motives individuals to try to prove aggravated harmful passions with them, in order to get more favorable annuity. In fact both effects should be avoided, on the one hand because they are directly deteriorative, on the other hand because selecting the potential fraud may steepen the annuity setting process.

¹² As a curiosity I mention, that many expert questions whether prevention diminishes health-care expenditure. It is possible that the National Health Service spends more on those having a healthy life style, because they show up at the screenings, and get treatment at an early phase of the illness, etc. While unhealthy lifestyle is often coupled with people not participating in screening, not acquiring treatment, they only go to see the doctor, when there is already nothing to do, so they are not so costly for the health-care fund. If it is true like that, discrimination according to the health status may be justified in the annuity system, though that would generate an incentive in the non desired direction.



5. When insurers traditionally distinguish according to age and gender, this is the result of considerate evaluation, they emphasize the factors, which have the most important impact on the life-span:

- It may be easily, reliably, costless surveyed
- Reliable data are available on the correlation between the parameter and the life-expectancy
- It is a clear and stable (calculably variable) characteristic of the insured

From this aspect the age and gender are incomparably useful parameters, than the ones earlier listed possible factors for differentiation:

- Accommodation, housing
- Workplace, nature of the work (e.g.: physical, intellectual, etc.)
- Health status (disability, illness)
- Lifestyle, luxury habits, sports
- Marital status

Accommodation, workplace, nature of the work, customs, marital status, habits, health status all change, respectively it is difficult to characterize them by a single parameter, to compare the different individual situations. It is even more difficult to survey them objectively (perhaps except for the marital status, there is a great opportunity for the concealment, for the presentation of the situation not in conformity with the reality respectively, so these data may be acquired by the insurer costly and non reliably (though to different extent).

From among the above listed the income (via this the expected annuity amount) and the self-selection are clearly, costless and well identified, nevertheless it is true that we do not have – at least in Hungary – good statistics about their impact on the remaining life time. During the lifetime the changes in educational attainment, but it is a stable factor relatively easy to identify prior to retirement, and there already are certain statistics concerning its correlation to the life time. (This latter applies to the marital status too.)

Despite of the above listed obstacles there are not only actuarial but some sort of “social policy” arguments in favor of differentiation – along certain parameters. In



this context it is important that the regulator takes into account that in the case of a mandatory insurance – for example the mandatory annuity – the prohibition of discrimination in premium also means that the regulator sets a forced solidarity among certain risk groups of the population, in other words it forces groups of people to financially support other groups.¹³ This type of redistribution is well known in other fields of the social life, but it is important to repeatedly call the attention that the usual ideology of annuities – according to what the state takes from the relatively better off and gives to the poorer, or that the common social cost is put to a relatively greater extent to the richer than to the poor does not fully work, to a certain extent just the truth is just the contrary. The relatively poorer have shorter life time, so they support the richer of expectedly longer lifetime. Only the lack of differentiation between man and women contradict to this, regular arguments in favour of redistribution here mainly continue to be in place. This phenomenon is often referred to (not “value free”) in the technical literature as “pervert redistribution”.

In the context of the pervert redistribution we have to mention, that the prohibition of discrimination in men, women annuities – generally – works against it (while it maximally effects a “non-pervert” but we may say “usual” redistribution), since it renders women of generally less favorable condition in the labour market into a more favorable situation during the annuity. We can not discriminate let us say according to educational attainment either, clearly the layers of higher income get into a better situation compared to the ones of lower income, therefore in the case of mandatory annuity the extension of the prohibition of discrimination on all every other factors beyond age may not necessarily be well justified. Let me mention here a factor to be detailed later in the study that the lower the technical interest rate, the annuity is more favorable for the higher income (because of longer lifetime) layers of the society than for the lower income (of relatively lower life time). Similarly the guarantee time is more favorable for the relatively lower income people. In summary

¹³ Adopting the view generally accepted in the actuarial profession I do not consider insurance itself, respectively financial transfer necessarily taking place among the insured via the insurance, solidarity. I do consider this character of insurance natural. It does not include neither force nor solidarity. However solidarity appears in insurance, and in the form of forced solidarity, it groups of evidently different risk and of balanced premium are classified into one risk community. The magnitude of the forced solidarity is precisely as much, as much money is transferred by one risk group to the other due to the prohibition of discrimination.



the pervert redistribution may be diminished by the case when annuity is discriminated according to a factor well correlating with the income status (e.g.: educational attainment) too, or the fixed, but variable technical interest rate is applied – to be discussed later, respectively we apply a buffer-type death service as well – also to be discussed later.

2.1.4. Options for the legislator

For the avoidance of the fraud I would suggest to calculate only with the factors from among the above mentioned ones that can be surveyed relatively reliably and cheaply, therefore the following options for discrimination may be primarily mentioned:

- According to gender
- the magnitude of the pension assets
- health status
- certain occupation
- educational attainment

Beyond these self-selection due to options made among different types of annuities with different mortality tables) that is, discriminations according to the option, the theoretical impossibility of which will be proven in the coming chapter.

Discrimination **according to gender**, as a possibility of mandatory annuity shall not be considered for the time being – based primarily on political and not technical arguments. This raises significant technical problems, what may possibly facilitated by other measures, for example by the mandatory double annuity – to be discussed later. Nevertheless there may be such a technique by the help of which the arguments against discrimination according to gender may be neutralized, and this the regrouping among individual accounts, on the impact of which the magnitude of the annuities of women and men is identical with what is calculated on the basis of a unisex table, while the actual calculation is made on the basis of tables different for men and women. The mechanism of the solution and related arguments were already detailed in a previous article of mine (Banyár [2002]). Among others I have detailed that the unisex mortality table, what is clearly in favour of women contrary to men, compensates certain inequities against women, but despite of this I do not approve its



implementation. In the very same study I said, that is even if we stick to the idea that women and men of similar age receive the same annuity for the same savings, it would be much more correct to directly regroup the assets from the pension account of men to the one of women in the magnitude that finally it simulates the situation that presents itself by the application of the unisex table, than to apply the unisex table.

If this regulation does not homogenize the magnitude of the possible annuities, the **discrimination according to magnitude of the pension assets** presents itself. This – to a certain extent deals with the differences in accommodation, physical - intellectual work, educational attainment, and lifestyle. Though we have to note that we have no data about the rate of positive correlation between the expected remaining lifetime and the magnitude of the accumulated assets in the private pension fund supposed by many and by myself as well. These should be systematically collected (provided, but not allowed that these data do not indicate correlation between the two factors, as it is supposed by some experts, or possibly these would indicate negative correlation).

On long term the implementation of **differentiation according to health condition** is worthy to consider as well, though for the foreseeable future (as long as it is taken into consideration in the health care system) it is useful to avoid that characteristics widely considered negative (basically harmful luxury customs, smoking and alcoholism) assure advantage in annuities (even if it would possibly justified from risk point of view). Differentiation according to health status may be very much different; practically we face a group of factors, which may be taken into consideration gradually as we learn more about their impact on life expectancy. First the impact of the well documented disability may be considered, it is important to note though, that the structure of disability pension and its adjustment to the system of private pension benefit is an important problem that I do not deal with in this paper.

Taking into account long time spent in certain occupations - e.g.: miner - as life expectancy factor may also be raised. However two important issues shall be mentioned in relation to this factor:

1. The trend of technical development is clear, occupations requiring physical effort gradually disappear so this factor of differentiation loses importance on longer term.



2. In Hungary the actual pension system significantly abuses this factor, because it defines a number of occupations (armed forces, dance artists), where many years of retirement-age benefit is assured, what is not justified at all based on the actual acquiring so that can only be interpreted as a political heritage. On the agenda we may have the gradual elimination of existing ways of discrimination rather than taking differentiation by occupation into account. In Hungary these sort of discrimination officially only exist in the pension system in Pillar I, the act does not provide whether these must be taken into account in Pillar II and if yes, how. This obviously generates a tension between the two system when the annuity phase starts.

Differentiation according to educational attainment meets most of the necessary requirements: it is easy and cheap to survey, “politically correct” that is it favors the poorer against the well to do ones, statistics are available about its impact on age, and these indicate that according to this factor the remaining lifetime of the Hungarian population is extremely diffused. It is useful to examine the application of this factor among the first ones especially its discrimination by gender can not be implemented at all by any method. Naturally such technical questions must be first answered e.g.: until what time may the educational attainment as a factor being taken into consideration change in the course of calculation (e.g.: until the annuity starts or perhaps until a certain fixed age).

Similarly to the differentiation according to occupation on longer term the same concern may be raised here, namely this factor would also run out in the coming decades. There seem to be a trend nowadays that higher education becomes ever general. This emphasizes the requirement of differentiation that we have to leave possibilities for differentiation according to other factors open for the future.

2.1.5. Differentiation by premium and provision

Above I discussed differentiation as discrimination by premium. Discrimination by premium is obviously followed by differentiation by provision, it may be logical that it is a more elevated method (by several factors) than the one by premium. This may specifically be raised when the regulation – due to different, non technical considerations. Basically prohibits differentiation (e.g.: by gender and age and by any other factors).



This may lead to the perception that two annuity portfolios of strictly different composition from risk point of view appears to be identical and the same provision is created for them what would not be right from prudential aspect.

Theoretically any parameter may be used for differentiation by provision, no considerations such as premium differentiation by gender, are raised against it. It is logical that even when using unisex premium table the gender of insured be taken into account in the course of generating provision even if the same amount of premium was collected on them than on the portfolio including more men.

In the course of provision generation differentiation according to other factors beyond the above would be useful with respect to prudential aspects. The highest education as factor to be dealt with is the best from among the above ones (say the population may be classified into 2 or 3 such groups).

2.1.6. Differentiation in the technical literature

Blake [1999] mentions “adverse selection” and mortality risk as major problems of annuity for providers and annuitants. On the basis of asymmetric information only people expecting long life volunteer for concluding annuity contract. According to Blake the insurer is unable to differentiate among annuitants of diverse risk. Therefore the annuity is calculated on the basis of the mortality of a select group so for those of worse mortality, annuity would be a bad business.

Blake’s statement contradicts to what I stated above, because in my view the insurer is able to differentiate, as it is indicated by the recent developments of the annuity market, which was not so much obvious yet, back in 1999. I agree with Blake to the extent (as I have written) that insurers have not differentiated among the annuitants until quite recently, what is probably the consequence of the small voluntary annuity market, therefore it is little attractive for the service providers. Annuitants were offered a high premium as a consequence of the above what further restricted the market. So this phenomenon existed, but in my view Blake explained incorrectly and not everybody agrees with it in the literature either, because in a competitive market better premium is offered for those of better risk according to James [1994]. Finkelstein-Poterba [2002] traces back the phenomenon to the decision of the insurers, according to them insurers do not collect data on social-financial status



and they do not differentiate in premium accordingly what causes loss for the low status insured who buy annuity. Insurers thought this statement need explanation and according to them the plausible explanation is that verification of such data is expensive. Based on a survey when information on annuity premium was collected from many annuity brokers it turned out that provider do not much differentiate among clients, what's more they do not specifically prepare to differentiate in the future because to set the premium they were only interested in age, gender, type of annuity, type of the market (voluntary, mandatory), assets, frequency of annuity payment, duration of warranted period and the service for the possible survivor. There were only a few companies to offer specific premium for client in bad health condition e.g.: smokers.

The first problem with the private annuity is this adverse selection according to James [1994] too. As Finkelstein-Poterba, he also mentions that for the insurers the price of the annuity is actuarially not fair for the good risks (that is for those of shorter than average life expectancy), so these products are only bought by the rich ones, in other words the annuity is a luxury good. James only mentions but does not explain the phenomenon, though calls the attention for its consequences the redistribution among the annuitant. This already appears on the voluntary annuity market, where in principle there is no obstacle to provide annuity of preferential premium for those of higher mortality. This redistribution may pose a special problem, where prices can not be differentiated; what's more if annuity purchase is mandatory at these prices. According to James [1994] such uniform prices mean ex ante transfers from the low risk ones to the high risk ones. It is particularly unfavorable for the low income workers. This means it is not a usual redistribution, because in this case the money flows from the lower income (therefore shorter life expectancy as an average) people to the higher income (therefore longer life expectancy as an average) people. Consequently this sort of redistribution is generally called "pervert" in the literature.

Redistribution by prohibition of differentiation was examined by others too. Brown [2003] came to the conclusion that in the mandatory annuity if the big redistribution item happens at uniform premium it may even be worthwhile for layers of high mortality rate on the basis of utility function if the cost of annuity administration is low. It seems that Brown got these results with disregarding the



bequest motive because at the end of the study he called the attention that in the future researches this impact shall be taken into account despite of the fact, that there no consensus among economist as regards the significance and the modeling of this motive.

In the Hungarian literature the issue of redistribution was raised in the context of private pension annuity in such a form that efforts shall be made to avoid it. In the study by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] interestingly it appears not as a result of claiming differentiation but in the refusal of Guarantee Fund and the yield-balancing provision, as redistributive measures, what is contradictory to the so often quoted basic philosophy of the private pension system – self care individual savings, etc. (page 486) (I do agree with this as it will be later elaborated). The study focuses on non transparent character of these redistributions as a further negative element. Michaletzky (Michaletzky [1999]) also agrees with the avoidance of redistribution and this basis he find the implementation of unisex table problematic (page 103). “The most neuralgic point of buying the service is the implementation of the adequate mortality table. [the unisex table] is clearly such an item of the act, in the course of application whereof the principle of pure fully funded fund is compromised and – appreciated – social elements appear.”

In the literature the issue of differentiation appears primarily in the context of mandatory annuities. James [1994] says if the annuity is mandatory an important public policy issue concern is according to what factors premium differentiation can happen. Can it income, gender, race or other factors? He mentions that for example in the United States of America race is a non prohibited factor of differentiation in employer pensions and in life insurance. (In the United States race is an important factor, in Hungary the eventual implementation of race would not even appear.)

Neither literature in English nor in Hungarian gives orientations as regards the issue of differentiation; along what criteria should this be made (e.g.: Michaletzky [1999] and Réti [1999]). They use a special wording: they say that it is not clear who create a risk community. This wording though mixes up the problem of differentiation and mortality profit distributon. (E.g.: when Michaletzky [1999] enlists the options of risk community he mentions the following: according to type of annuity, investment portfolio, age cohorts. He says it is not clear whether the fund is entitled to define who



belong to a certain risk community or it is defined by the law.) Réti [1999] thinks that every member of the fund is the risk community, active and inactive should be together, what more suggests the distribution of mortality profit than differentiation. There is one more or less majority stand as regards differentiation, this is a negative statement: differentiation according to gender must not happen – though some – as we saw at Michaletzky above - find it problematic. In the Hungarian literature of demography – irrespective to annuity issue - Ágoston-Kovács [2007] call the attention to certain researches, what may even be aspects of differentiation. According to this (Kovács [2007] page 7) “It is long known that mortality of widows and divorced is higher, the health status is worse than their peers living in marriage. Disregarding some exceptions worthy the attention similar regularities appears among unmarried and celibate.”

According to Winkler-Mattar [1999] in annuities the solution for the problem of the adverse selection is the differentiation of annuitants, though he mentions as a new aspect, that if the annuitant portfolio is differentiated, e.g.: more favourable premium for the population of higher mortality, which in principle would aggravate the problem of selection at the remaining population. Fundamentally this the problem of the old providers, at the same time this is a possibility for the new ones, as it is mentioned by Ainslie [2000] in respect of impaired annuities provided for the ones of shaken health status, accordingly these allow the new entrants being selected from the existing ones. According to his data the first such annuity (the so called nursing care) was sold by Eagle Star in 1991.

While in the voluntary annuity this selection is partly a possibility, at the same time it is a danger too, it depends on whether we speak about a new or an old service provider, while in the case of the mandatory annuity it is clearly a danger for the regulator, for this James [1994] already called the attention. According to him, if risk categories are created the annuity providers may skim the best risk in every category. Therefore he has a clear stand, that if purchase of annuity is mandatory the service providers must be banned from selecting among the annuitants or a specific fund shall be created for the coverage of the bad risk (the very healthy people). This must be subsidized by either the annuity providers or by the state. As for my part I think that this second solution is not the way to go, because it makes the system unnecessarily



complicated or it obviously transforms the money of taxpayers to profit at certain providers.

2.2. THE ISSUE OF THE APPLIED MORTALITY TABLE

There is one certain thing about mortality table used at annuity premium calculation and provision creation: the projected mortality table taking into account the foreseeable change of mortality must be used, tables completed of history data can not be satisfactory. The above factors of differentiation should also be considered by the mortality tables. An other issue though whether the central mortality table or the one used by providers shall be used, whether they depend on other elements of the annuity system.

2.2.1. The preparation of the mortality table, mortality projection

The question is who shall prepare (unisex and differentiated, but anyhow projected) mortality tables for annuity premium calculation, for provision calculation respectively. There are diverse possibilities. The more important ones are:

- A state institute established for that purpose
- An existing institute, (e.g. Central Statistical Office Demography Institute) or and institute entrusted with this task
- The concerned providers (life insurers and fund) or an instituted established by some of the providers
- Every provider to prepare these tables by themselves

The Central Statistical Office Demography Institute prepares mortality tables (e.g.: break down according to residence, educational attainment) but restriction to private pension fund membership is evidently extremely specific. There is good reason to assume, that life expectancy of private pension fund membership is different from one of the entire population of Hungary, but in the early phase (annuity calculations in respect of voluntary members) this difference is difficult to estimate. Later the difference from the entire public mortality table is probably gradually decreasing.

A single provider can only prepare mortality tables for itself validly if it has quite large, stable and old stock of annuitant. Since there is no company so far to provide annuity, nobody has sufficiently old stock. The stock may be regarded as stable, if the



fund members' possibilities to change providers are restricted, what would be a big step back compared to the actual system. Many providers have sufficiently large stock, but not everyone. Due to all these, nobody from among the actual Hungarian providers can be supposed to be able own mortality tables, on long term only some of them, therefore this option – at least for the time being – can be rejected.

In summary either we have the existing or a new institute the owner may be either the state or the concerned providers. Either way is feasible, but it is useful for the state to be in initiating role – based on provisions of the law – or by establishing, respectively assigning such an institute, or by assigning responsibility on the providers and setting deadlines for them. That is the mortality tables, the mortality projections shall be prepared centrally (respectively projections shall be regularly checked and recalculated), for it is useful to use mortality data of private pension fund members, what is already collected by PKN (Central Registry of Pension Funds) operated by HFSA (Hungarian Financial Supervisory Authority) (though it is possible, that the content of data shall be extended function to the potential factors of differentiation).

2.2.2. Central mortality table or mortality tables by providers?

It is to be decided whether the centrally prepared mortality table must be obligatorily used by every provider or not (if there are more providers at all).

In fact the following possibilities are given:

- Everybody shall mandatorily apply the unchanged, centrally prepared mortality tables
- The centrally prepared tables must be applied, but no justification is need for any diversion to a previously agreed extent
- Diversion from the central tables to any degree is allowed, but the provider must justify the reason for diversion
- The central table is just an aid for the provider,

The question may be raised whether the same applies for premium calculation as for provisioning from the above options. To illustrate it by an example: for premium calculation everyone shall apply the same central table in unchanged form, or the central table is mandatorily applied at provisioning though diversion from it is allowed at premium calculation.



No free option is allowed from among these, the choice depends on the other elements of the system. For example the latter option supposes that service providers take all responsibility for the calculation and provisioning, if they make a bad decision nobody would help them. This is a risk for the clients too. This may only be expected from the service providers if they may count on a stable range of insured, that is if clients can not freely move from provider to another one.¹⁴ If according to the annuity regulation insured may move from one provider to another one in the annuitant phase as well, it is important to clearly set the magnitude of the contingency they may take with them, this is only possible if the mortality tables used at provisioning are centrally defined.

When rendering the application of the centrally defined table mandatory it is important to take into consideration that such a requirement makes the state responsible if the application renders the anyhow state owned service provider get into a difficult situation. However the state may not necessarily save being responsible by not making the application of such a table mandatory if the state does not create the other conditions adequate for the provision of calculable annuity for the providers.

Making the application of the central table mandatory has significance for consumer protection too, since it prevents that clients get to irresponsible service providers who intentionally claim low premium and have low contingency, and after a certain period of time are unable to stand for the undertaken service.

If we take that the application of central tables are made mandatory at generation contingency or at calculation of premium, one may say that it is sufficient to require mandatory application of central table when generating contingency, and not to have any requirement at premium calculation, it is simply the appearance of free premium calculation.

¹⁴ This is the reason why I find the actual regulation concerning this option deeply problematic. As item (1) Article 6 of Kr. 170/1997 provides: the mortality tables defined in (1) Article 32 Mpt as well as the ones to be used in (2) Article 16 shall be selected or prepared from the mortality tables published by the Central Statistical Office by the actuarial of the fund with respect to the demography conditions of the fund membership receiving the service. Finally this places the burden of all responsibility concerning the premium calculation and contingency generation on the fund, the actuarial in reality but with the situation that it does not stabilize the stock of insured at the fund, does not permit to adequately make the calculations. So this regulation is contradictory in itself and is worthy to change.



If the central table is required at the premium calculation too, this explicitly emphasized the responsibility of the state for the adequacy of the calculation, what only works if the state operates a premium blending mechanism parallel with imposing the mandatory application of the central table. The imposition of the central table at premium calculation (and here we only mean the calculation of net premium) is coupled with the technical interest rate centrally defined gives the advantage for the client that they can easily compare the premium set by different providers. Though it is basically the case even if the central table (and the technical interest rate) is imposed only for the provisioning, since this already determines the premium calculation too.

Transitory solutions between the application of the mandatorily defined table and between the free choice is not really a clear solution, one can not argue in favor of it. These transitory solutions may be applied in such cases when the state wants to regulate, but at the same time wants to get rid of the responsibility for example in such a way that it does not give tables of parameters differentiated by adequate amount of parameters, so responsibility along with the accommodation to the given risk community, to its exact composition if finally left for the provider.

Based on the above we have the following possibilities:

1. at provisioning every provider is obliged to apply the central mortality tables (differentiated according to several factors, primarily by gender and educational attainment too), but at premium calculation they may freely divert from the central (unisex) tables.
2. the central mortality tables must be used both at generation of contingency and at the calculation of net premium

Method No.1 seemingly gives freedom at premium calculation, at the same time it allows providers to accommodate to the expected risk composition of the risk community. It also signals that responsibility for the bad composition of the risk community is transferred by the state to the provider what is not necessarily an equitable method, if differentiation according to the above important aspects is prohibited.¹⁵

¹⁵ By this the composition of the risk community becomes incalculable, because let us say, that the provider expects a lot of many, therefore it sets a relatively low premium, as a response a lot of



In the case of method No. 2 the state shall explicitly recognize what is implicitly recognized by the imposition: it takes responsibility for the adequacy of the calculation. In other words – as I have written above – we explicitly recognize the responsibility of the state for the adequacy of the calculation, and parallel with the imposition the state operates a premium blending mechanism. Prohibiting the differentiation gives strong arguments to support the operation of a premium blending mechanism, since it is not certain that the failure of a provider is not the responsibility of the regulator (by the prohibition of differentiation).

If the state imposes the application of table differentiated by adequately high number of parameters it leads automatically to the issue of (characteristically closed) funds of specific composition, these do not require a special treatment. If the tables are not adequately differentiated, it is possible that the closed funds must be separately managed – however these must be made closed from every aspect, that is members must be prohibited to opt for the provider.

A specific question is the groups of specific management in Pillar I, (policemen, prison guards, soldiers, dance artists) but I will come back to this issue.

2.2.3. The issue of mortality table in the technical literature, the unisex table

Literature starts enumerating problems concerned with mortality table with the fact that no reliable mortality data are available in many countries; fortunately Hungary is not one of them. This however is a major obstacle of annuity market development (e.g.: Rocha-Thorburn [2007]). Nevertheless not all the necessary mortality related data are available in Hungary either, as it is mentioned in papers written by Stahl and Michaletzky when speaking about the development methods of unisex mortality table, also they give details of what further factors shall be taken into account at an adequate annuity mortality table.

In the case of annuities with special regard to “adverse selection” different mortality table shall be used than in the case of other life insurances (e.g.: Hylands-Gray) [1992]). They say that for the mandatory pension annuities the use of a single basic table is justified, in this case selection effects shall be not calculated with. Mehr-

women would come here leaving behind providers expecting a lot of women, by this the calculation turns upside down, and there is an immediate loss at start at the provider.



Gustavson [1987] states (page 533) that in case of annuities mortality table different from the ones applied at other life insurances must be used due to three reasons. No. 1 the mortality tables used at life insurances includes a security factor (in other words a worse than actual mortality is assumed), what is a life danger for the provider of an annuity of opposite risk. No.2 There seems to be a trend for decreasing the mortality probability. At normal life insurances this even increases the safety for the provider, but in the case of annuities it causes loss. No. 3. People in bad health status do not buy annuity, so those who buy annuity belong to selected populated segment.

Unisex table of mortality is dealt with by the Hungarian and English language literature. According to the previously quoted Mehr-Gustavson [1997] page 533-534) there is an increasing demand for the unisex table in the society, especially in the case of employer pensions. In the case of individual annuities – as women buy other individual life insurances cheaper – it would be logical that annuities are more expensive. Despite of this there is more frequent request for the use of unisex table. According to the authors it seems that actuarial considerations are ignored at the heated social disputes, and the actuarial equality is replaced by social equality so it is possible that the unisex tables will be implemented at the individual annuities as well.

According to a more recent literature (Curry-O'Connell [2004]) every annuity is differentiated by gender in England, but this study examines the possibilities and impacts of mandatory pricing annuities by unisex tables.

In our opinion the arguments in favor of the unisex table are as follows:

- in the case of identical pension assets payment of non-equal annuity is discrimination
- with respect to the fact that there is a significant coincidence in those ages when most people die, the observed difference of the life expectancy of men and women is irrelevant
- unisex annuities would increase the pension income of women
- gender is increasingly a less relevant factor in annuity pricing

Arguments against are as follows:

- annuities differentiated by gender may not be considered discrimination with respect to the longer life of women so for men and women the expected income is equal



- In any year an insurer is more likely to pay annuity for a woman than a man, who bought annuity at the same age, so the coincidence by age is irrelevant.
- The unisex annuity would decrease the pension income and would increase expenses.

According to Curry-O'Connell [2004] a further argument in favor of unisex annuity is (and this is the most important one too), that no differentiation can be made between men and women in labor law. In the USA and in Canada the employer DB pension is mandatorily unisex. The unisex employer pension exists in the UK as well; this makes up some one third of the market. It is to be considered, that in the UK and in Sweden unisex annuity is provided in the state run pension system too. They say, that is unisex annuity would be mandatory, unisex premium would be better than the actual one, when it is voluntary. They think that despite that unisex premium would not have significant impact on the pension income; they did not find any argument against the unisex annuity either.

The Hungarian literature deals a lot with the unisex mortality tables, respectively the unisex annuity. Réti [1999] thinks, that the unisex annuity does not mean uniform unisex annuity in the entire market, only within one fund, therefore premium may be extremely different at funds with majority men and majority women. According to Stahl [2000] the two weak points of the private pension system are the normative annuity (it was abolished while this study was prepared) and the unisex annuity. In respect of this latter one the major problem according to Stahl is that the regulation provides the determination of premium at identical way, but the provisioning must be differentiated by gender. This leads to the problem that it is not certain that income and expenditure by fund will be met. In fact the balanced unisex premium is the average premium, that is the premium of men and women weighted by assets, and this is a value depending on the fund, that is the unisex premium is different by funds even if men and women in each fund die according to the national average. On top of it this value is uncertain because of changing from one fund to the other. According to him there is a solution for the problem of unisex annuity only if this problem of the different providers is eliminated. He may think of two different solutions:



1. annuity is purchased from the insurer for every man and the fund provides that for every woman. In this case the unisex mortality table is identical with the women mortality table
2. there is a single risk community with a single state run provider. The premium may be defined by using a linear software programming application. The consequence of the solution is that there is no need for a specific Guarantee Fund.

In the study written by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] (probably the annuity part was written by Stahl) repeated that the provision of the law poses a problem, according to this the premium must be set on a unisex basis, be contingency must be generated in a differentiated manner. According to the study the problem can not be solved by the regular equivalence principle what is provided by the act. The answer for the “why” is not detailed in this paper, but in the next study written by Stahl [2005]. Here he restates that the problem may be partly managed by a central provider, partly by the elaboration of the mathematical programming of the objective; according to him this is almost a completely different method than the application of the equivalence principle, but the difference between the two methods is not detailed. Therefore we depend on guesswork concerning the difference: perhaps according to Stahl the equivalence principle is not correct because calculating by any unisex table it is not clear that the total premium collected would be sufficient for the generation of contingency calculated with differentiated mortality tables. Therefore for the determination of the premium he wants to take into account the composition of the annuitants by gender, in addition he proposes weighting by their assets. In my view Stahl’s solution is problematic because:

- unfortunately there is no guarantee of any kind that the actuarial has preliminary information about the gender composition of the annuitants. If there are competing providers, it is certainly not known, if there is a central provider, there is a fairly good approximation, but no precise composition is known because the decision to retire is made by the individual, and the central provider will certainly not know about it in advance.
- On top of it people do not retire at the same time – while Stahl formally ignores time, but the solution may only be that he supposes that people



retire at the same time or at least in big groups. As this is not the case there is a doubt that is programming objective can at all be defined. So we are left with the equivalence principle.

- Approaching the problem from an other angle: if – using Stal’s assumption – we possess all data for the mathematical programming, this is certainly not needed, since this information may be incorporated into the structure of the unisex mortality table, and the equivalence principle would give completely good result.
- So as for my part, I do suspect that since Stahl has never been a practicing actuarial, he was not completely familiar with the actuarial methodology therefore he rather applied the methodology well known for him (mathematical programming) for the solution of actuarial objectives as well.

Stahl [2005] induced a dispute among experts; basically it is the single public debate about the issue of private pension annuities. Stahl’s statements were disputed by Miklós Arató in two articles, and György Németh also commented them, Stahl responded. From mortality table aspect only Arató [2006a] is important here. He thinks that prohibition to differentiate between men and women to overemphasized, since there are similar differences on the basis of smoking and non-smoking people, obesity and non-obesity, but nobody wants to differentiate according to these criteria. In fact his argumentation is a bit surprising, as he does not say what an average actuarial would, the more stable the calculation the more criteria are used for differentiation, but he find the existing differentiation too much because no other criteria for differentiation are applied. According to Arató (page 272) “It is completely irrelevant that good mathematical programming model was prepared for the determination of the unisex annuity is we are 20 - 30 per cent wrong in the determination of the remaining life.” This dispute was later referred to by Ágoston-Kovács [2007] so that in the Stahl-Aráto-Németh dispute on unisex table, respectively due to the differences between the life expectancy of two genders are in the centre. He also states that (page 562) “...there is still no solution for the definition of the private fund annuities, even there is no generally accepted proposal. The range of “solidarity” between men and women is not defined; we do not know whether the difference present among genders in respect of the life expectancy should be



equalized nationally or within one fund. If within one fund, that would generate unpredictable transfers among funds, as everyone wishes to get into a fund where there are more young men. Probably efforts would be made to keep women away from the fund, for example by the establishment of closed “miners funds”.

2.3. LONGEVITY, RESPECTIVELY MORTALITY RISK IN GENERAL

2.3.1. Differences between the theoretical and actual mortality

When above I have given the individual contingency formula of life annuities, I implicitly supposed that the actual similarity of the insured's mortality is identical with the presupposed one (with the one used for calculation and included in the tables). In practice the actual mortality will be different from the presupposed one, therefore mortality loss or gain gets generated at the providers. The cause of the difference can be twofold:

1. fluctuation of mortality from year to year, (random deviation)
2. mortality change is different compared to the one projected (systematic change)

The trend itself, if systematic mortality change is manifested, it may be called a longevity problem, what can be divided into twofold again:

The actual trend in

1. the stock of insured, respectively
2. at a certain service provider.

The entire stock of insured – independently from the type of differentiation allowed, purely from technical point of view – must be differentiated from risk aspect, and different mortality tables shall be established for the different groups, in those groups trends of different rate will naturally manifest. (For example during a certain period of time the remaining life of women with diploma grows much more than of men who attained maximum 8 classes of elementary school, etc.) In other words the projected and actual mortality trend of the entire stock of insured would differ from each other in different ways.

The mortality trend in differentiated groups of certain providers may be totally different from the trend of the adequate groups of the insured stock due to two reasons:



1. The given differentiated group at a certain provider is very small, therefore the actual trend of this small group randomly diverts from the “large” trend.
2. The composition of the insured at a certain provider is not random according to the characteristics ignored at the differentiation, that is it does not reflect the composition of the entire population, but systematically diverts from that (e.g.: alcoholics or those with awareness in nutrition are highly represented at the given provider).

Naturally the trend or the systematic change first appears as a random diversion (fluctuation), so these two effects can be separated from each other only on longer term. Though it is obvious that the fluctuation is the higher the smaller the stock of insured is, what is more, with an adequately small stock of insured the fluctuation will certainly take place, since the number of insured themselves and the number of deceased is an integer and not every mortality probability - fraction – can be generated as quotient of arbitrarily small integer. So vice versa the fluctuation of mortality from one year to the other decreases in proportion to the increase of the stock. There is an increasing probability then that the reason for the deviation between the theoretical and actual mortality is a trend. So this trend can be recognized and quantified in the shorter time the bigger the stock and vice versa.

I suppose that the trend of the entire stock of insured (respectively differentiated groups of them) will be identified and quantified by a central institute based on the collected mortality data. I can see a small chance that the trend manifesting at different providers can be separated from the accidental fluctuations, therefore in the following I start from the fact that there is only one kind of trend, the trend (of differentiated groups) of the entire stock of insured, and deviation from that by service providers is listed among random fluctuations. However we must take into account that the cause of different trends of the providers may finally be the regulation, the restriction of differentiation.

2.3.2. Whose is the mortality result?

Before discussing the management of mortality risk the question arises finally whom the right to the eventual mortality profit resulting from the difference between the theoretical and the actual mortality in the end. I say “in the end” because it would



be easy to give a mock answer, that to nobody, but it may be saved for the coverage of the later mortality losses. This is a mock answer because if we do so, mortality losses and profits accumulate on longer term, but in the end we have to raise the same question as regards this profit. Who is to be credited or debited? So I reject this answer and I only regard the accumulation of profit or loss as a possible method of problem management.

So whose is the profit that comes from the death of more insured than it was preliminary calculated? It is easy to answer the question by saying that naturally it belongs to the insured ones, since the annuity generates totally from their assets and the provider only redistributes this and the yield of it among them. This is a completely legitimate answer; however the question was somewhat misleading because I did not ask about the mortality result or the mortality loss, but about mortality profit. Should I raise the question so that whose is the mortality loss the same people who answered the previous question by saying it belongs to the insured ones, would most probably say that it is the provider's. Nevertheless it is obvious that these two answers are contradictory to each other, since why should the provider only carry the loss if it does not get any share of the profit. Clearly the consistent solution can only be that the mortality to whoever the mortality profit belongs would carry the mortality loss too. It must be seen that in the developed world, and according the trends¹⁶ persistent since the second half of the 90-ies in Hungary too, not the mortality profit, but the opposite, the mortality loss is more likely, though in the end it depends on the projection (if we project a too speedy mortality improvement, the mortality result may systematically be profit).

The mortality profit / loss finally¹⁷ may be credited / debited on three players:

1. the state/ the state (in other words on taxpayers altogether)
2. the annuity providers / the annuity providers
3. the insured ones / on the insured ones

The first item may be raised at all, because the state plays a role in generating of mortality the result as well, partly by prohibiting certain differentiation, partly by

¹⁶ This is called 'longevity phenomenon, what is in fact the unpredictable / incalculable increase of the life time.

¹⁷ if it is eventually carried over in the form of mortality contingency, then on long term,



preparing and imposing central mortality tables for the provisioning and mortality outcome may be the result of both of these. Though there is a consideration according to which if the state is involved, , in the financial matters of annuity even to a small extent, such state intervention may grow to an unpredictable magnitude, which suggests that it is useful to keep the state away from the annuity issue in financial terms. In addition should we look at this issue on any long term, loss and profit will probably have a balance different from zero. If it is positive, that is the state wins on it, one may question why the state withdraws the money of insured ones for other purposes. If it is negative one may raise the question why should annuitants be supported by some other tax money. So this solution is best to be avoided, though whether this can really be made depends on the implemented annuity model. If the regulator opts for the model of central service provider, such an intervention may much more easily happen – and to a certain extent it is much more justified – than if the providers are market players independent from the state. Nevertheless even in the model of central provider it is justified to endeavor that the state remains neutral in financial terms in the annuity system.

If the annuity provider is a profit oriented enterprise of which the insured is a client, it may be logical to expect that it protects the client from a decrease in the annuity due to the mortality loss, that is it is expected to take over the mortality consequently it will be entitled to the profit too. There may also be an expectation, that mortality profit accumulated on longer term is not exaggerated, so that the provider does not withdraw much from the assets of the insured for its own profit (keeping the long term accumulated mortality loss low is the interest of the provider who shall assure it). If the annuity provider is a non for profit enterprise (fund) owned by the insured, there is nobody else, not even in principle, but the insured to take over the loss. In this case it neither arises that the provider bears the loss, since the burden of the loss falls back the insured via the provider and the profit belongs to him too. Though a solution may be imagined, where the mortality loss is born by a range of fund members different from those who “produced” it (and vice versa in the case of profit), but it is clearly inequitable, therefore such a solution shall be avoided. Debiting the mortality loss of the annuity dues of the fund on the members who still are in the phase of accumulating assets shall be avoided (neither to assign mortality profit to



them). Highly avoidable the asymmetric therefore increasingly inequitable solution to debit the mortality loss on members still being in the asset accumulating phase and annuitants retain possession of mortality profit. This solution is to be avoided, because such inequitable regrouping makes membership shaky, by this being given arguments for changing into funds of more favorable demographic status. Therefore the treatment of this problem might raise the abolishment of free choice of fund; it might start a “vicious circle” (“a regulatory spiral”).

Based on the above the mortality outcome may be born by the insured person himself, or the profit oriented provider (if the provider is such a person), or the outcome must be shared between the two of them somehow. To a certain extent the “purest” solution is if the insured person stands for entirely or partially mortality outcome, because in this case the question raised with the state and with the profit oriented provider is not raised, that too much cumulated mortality loss and profit are problematic, since both of them are owned by the interested person, that is the insured one. To whomever it belongs, one has to make efforts (that is such mechanisms must be operated to assure) that mortality outcome is close to zero on long term, otherwise either the insured person or the provider, nor different groups of the insured suffer regrouping; what should be avoided as it is inequitable.

Due the above arguments I do suggest numerous solutions for the management of problems concerning mortality outcome; these are valid if the mortality outcomes belong basically to the insured people.

2.3.3. Managing the mortality risk

Mortality risk may be managed by the following methods:

1. Efforts to take into account of potential mortality changes (trends) at each differentiated sub-group (with regard that differentiation at the provisioning is made according to several factors).
2. The mortality loss of the given year decreases the yield of the given year (at 0% technical interest rate, at higher technical interest rate than that), the mortality profit increases this yield that is it becomes part of the indexation of mortality outcome
3. Provisioning to smooth the lengthened longevity than the projected one



4. Requiring guarantee capital from the providers (exclusively from those having actual owners)
5. The risk is adjusted to the financial muscle of the different providers so, that the mortality risk is split into parts and we separate the normal and the peak risks.

Homogenization – to be discussed later – contributes to the management of mortality risk, so that one differentiation factor (the magnitude of the annuity asset) is cancelled (or its potential impact is significantly decreased).

Let us look at these methods.

2.3.3.1. Forward estimate of the mortality

I have earlier dealt with the issue – though far from satisfactory extent – and I am not going to indulge in the details. Projection of mortality is a complex issue, which has large literature, what I am not going to detail here. However as part of the “infrastructure” of private pension annuity either a new institute or an existing one must routinely deal with mortality projection. As I mentioned projection does not try to seize the annual random fluctuations but the long term mortality trend.

The inevitable part of the process is the feedback; mortality projections must be regularly controlled and updated by the actual data. Projections must be prepared for the entire stock of insured people as well for the different risk groups as well. In harmony with the above differentiation is worthy to be done in the following breakdown (while data must be gathered according to other aspects as well, in order that later the population can be broken down to further risk sub-groups):

- By gender, in the following breakdown
- Educational attainment (at least with elementary education as a maximum, secondary school graduates, higher education graduates)
- Magnitude of pension assets (e.g.: in such a break down at the age of 62 átlag)

We may raise the question whether how often is the projection corrected. I think that even on long term, not more often than once a year, since during shorter period of time probably no meaningful change happens, and the eventual annual cyclical fluctuation of mortality would make projection more frequent than a year misleading.



2.3.3.2. Mortality outcome as part of the indexing

If the mortality outcome – or a part of that – becomes part of the indexation, this means that it is either annually, directly transferred back to the insured people, respectively booked on them (debited or credited). This solution may be more or less forced (primarily in the case of annuity model where the providers are funds of our days, that is institutions without capital), or it may be a voluntary option. I have already elaborated on arguments in favor of voluntary option. The option of the solution depend on the foreseeable magnitude of the fluctuation in mortality, on other words how big the mortality outcome is expected to be. This depends on the size of the risk community. If there is a small risk community (e.g.: at a small provider, or at any provider at the start-up of the system, the provider respectively) this should not be used, because the fluctuations are very big. In the case of the possible biggest risk community – that includes the entire country – for example at a central annuity provider, or in the case of annuity model with a central annuity pool, its use is almost mandatory, or at least there are very few arguments against its use.

The joint treatment of the investment surplus yield and the mortality outcome tries to smooth the annual mortality fluctuations, which are partly random (respectively are due to the small stock) and partly are the result of the deviation from the trend, and that can only be separated from the random fluctuation on long term.

The mortality outcome may be either profit or loss. It seems that there is no specific problem if the mortality profit must be distributed back to the insured people. However the mortality loss is a different issue, that the treatment of profit and loss is not necessarily symmetric. We may raise the question to what extent the investment surplus yield bear the mortality loss should, or in other terms: what portion of that is to be debited immediately, directly on the insured people. The question is rather raised whether the annuity may decrease from one year to the other due to the mortality loss, if no adequate surplus yield generated to cover the mortality loss in the particular year. (Naturally the question is raised not only in this form, but also as whether due to an eventual negative investment yield the annuity may decrease– this will be dealt with in relation to indexation).



The majority of insured people would probably prefer not to have any nominal decrease in the annuity for any reason, for example due to the mortality loss. In such a case the adequate part of the loss is either carried over (hoping for a better situation, when from the investment and/or mortality profit the actual loss can be recovered), or it is absorbed by some sort of provision. One may ask whether this provision is created by the money of the insured people or whether this should be the solvency capital of the provider. In the event of carry over two questions are raised? Until the loss is recovered what would be the source of the funding, respectively the previous question is raised here too, if the loss is not recovered, finally who shall absorb it, the insured people or the provider? In summary the problem can be treated as if mortality outcome would not be at all part of the indexation that is with the provisioning and the solvency capital. This also means that making the mortality outcome part of indexation in itself is not sufficient without one of the two other methods, so the methodology must be complemented by one of the two.

Making mortality outcome part of indexation does not mean that the investment and mortality outcomes are separately presented, and that indexation in summary is the result of which factors. Namely this prevents that the eventual mortality losses hide the bad performance of the provider on the field of investment yield.

In fact indexation based upon the combined investment and mortality outcome deteriorates the mortality outcome. Though it is not self explanatory in what range should it be used? The major possibilities are:

1. among the annuitants of all the providers
2. among all the annuitants of one provider
3. among annuitants of the same risk group at one provider
4. among annuitant of identical age at one provider
5. item 3 and item 4 together

Application or applicability of solution No. 1 depends on the annuity system The major possibilities are:

- if there is only one central provider, this solution is obvious, though dispersion according to the other possibilities is raised here too
- the essence of the central pool model is (later I will present it) that the mortality outcome is centrally managed, so it is dispersed in the entire stock



of insured. In this case not entire combined investment and mortality outcome is dispersed, only and exclusively the mortality outcome is dispersed. The central pool can be organized so, that certain elements of the entire annuity system (e.g.: annuity payment, etc.) is centralized, and also in such a way that it operates only as a kind of risk balancing system. Each is a complicated system but the review is easier if several things are centralized, so if it is only a solution operated as a risk balancing system it is probably not easier, but more complicated.

- Voluntary pool may also be organized, and naturally here we may also think about the dispersion of the mortality outcome.

The reason for the central pool is, that it compensates the providers (and insured) of bad composition for the loss due to the composition, what may not be treated by differentiation due to the regulation. Its complexity is against it.

Solution No. 2 is self explanatory, if the range of insured people is fairly restricted, so it can not be further split. We may say that in such a case the provider can not do anything else, but opt for this solution. However in case of a big stock of insured people (beyond a certain threshold) it is more correct to further break down the stock that is to opt for solution No.3. If the stock is even bigger (so beyond another threshold) it is worthy to distinguish among cohorts (that is to apply solution No. 4 and 5 together), since in the mortality a generation impact can also be seen, and it is justified to say, that each cohort shall bear its own risk.

Solution No 4 may still be mentioned as a specific version, a diversion among annuitants having identical annuity starting year, but provided only if the annuity model is such a version of the central provider, that in certain years contracts may be concluded with a single provider (with the one that won the application for that particular year), and with another one in another year. Therefore the risk community by providers here is split according to year of the retirement and this is mostly identical with the splitting by providers, that is solutions No. 4 and 2 overlap.

Theoretically it is possible that such loss is transferred to the later annuitants via increasing the premium, but is a totally incorrect therefore indefensible solution.



2.3.3.3. Provisioning to smooth the fluctuations and to elongate longevity

If it is afforded that the entire mortality loss of a year is covered by debiting it on the investment yield irrespective that the annuity may possibly decrease due to this, there is no need for specific provision to smooth the fluctuations. But if we want to restrict fluctuation (either so that the annuity may not decrease from one year to the other, or so that the increase of the annuity may not decrease below a certain level – e.g.: below half of the inflation) from one year to the other, then there is a need for provisioning or for solvency capital. Provisioning and/or solvency capital may not only complement but also replace that if the mortality profit is covered by debiting the surplus profit of course it has a price.

The provision may have two sources:

1. a portion of the premium paid for the annuity
2. a part of the surplus yield of regular investment

The source No.1 will probably not be enough, either too much is generated of that because at the beginning of the insurance this can not be correctly calculated how much will be needed during the period of time. Therefore this source may not well be used independently from source No. 2. On the other hand source No. 2 must fundamentally be used to the extent, as the mortality loss occurs, therefore this solution indirectly leads back to the solution that the mortality provision is treated together with the indexation, therefore it should be considered whether the application of the indirect method would be better or not.

As regards provisioning let us see what sort of fluctuations shall be smoothed. The possibilities are as follows:

1. only those that are the consequence of mortality fluctuation, though for the insured people it is a too abstract solution.
2. those that result from the joint fluctuation of mortality and yields. The essence is that in years of “good yield” the annuity is intentionally increased at a lower rate, by this we can save for the years “of bad yield” when annuities can be raised to a higher extent than the yield. The danger of this solution is that it enters a voluntary element into the system, because we can never be certain when the “good year” is and when is the



“bad year”. These are relative terms; we can certainly judge it with posteriori, when it is too late. It may easily happen that bad years accumulate, and a “bad year”, when the provision was already used, is followed by an even worse year.

3. only those, when the mortality outcome and the yield together would bring the annuity below a certain level (e.g.: below the annuity level of the previous year). The advantage of this solution is, that a severe, clear rule prohibits the voluntary “smoothing”. Naturally circumstances may continuously evolve unfortunately even at this solution, and due to this the provision may become exhausted (despite, that it is reasonable to construct the system in a way, that the provision is topped-up at the earliest possibility). In this case there are two options: either the clients absorb the loss (that is the annuity decreases) or (if there are actual owners) it debited against the solvency capital.

There can be two sorts of provisions:

1. As a safety part generated within the individual provision
2. Separated from the individual premium provision (and from any other provision) as “demography” or “demography and yield balancing” provision, what – compared to the other possibility – may be called “collective” provision too.

As for my part, I would clearly prefer the first one, due to the following.

We have to note, that the regulation included the “yield balancing provision for a long time, its criticism appeared also very soon. For example the study by Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] says (page 486-487): “Provisions diminish the magnitude of the service, the redistribution due to the provisioning is contradictory to the self-care, saving, the so often quoted principles of the private pension fund system. The yield balancing provision properly illustrates the non transparent and unnecessary regrouping. The roughly even yield rate during many years is a questionable objective, as the real target should be to have the possible largest balance of the individual accounts at any point of time. The yield balancing provision is certainly contradictory to such a target. ...the only consequence of the



provision is the redistributions, what is a complicated procedure from implementation point of view.”

2.3.3.3.1. Individual provision – buffer in the annuity

This solution promises a conditional extra service to client. The condition is that due to unfavorable yield and/or mortality loss there was no need to use the coverage of the extra service for maintaining the annuity level prior to the maturity of the service. If the coverage of the extra service partially decreases because of the above events (what may be regarded as a kind of “calculation surplus” or safety supplement” or “safety valve”, or “buffer”) then the service itself proportionally decreases at maturity; if the coverage completely runs out, prior to the maturity of the extra service there was no possibility for topping-up, this service is completely cancelled. If the coverage of the extra service diminishes as is it was acquired, then in the first such year, when the total yield and mortality outcome is positive, from this outcome the individual provision must be topped-up to the necessary level of the extra service (so the indexation of the given year may be only as much lower). The total yield and mortality profit, and even the profit of the consecutive years, must be entirely use for this top-up if it is necessary, until the extra service is completely covered (until the entire coverage is available for the smoothing). If the provision within the individual provision completely runs out and a bad year comes again, then this must either be absorbed by the insured – depending on the regulation) – in other words the annuity of the insured person decreases, it may happen at a provider without solvency capital, that is at a fund; or the for profit provider covers it against the solvency capital. In summary at this method – contrary to the method of collective provision – the client does not lose the right for his own money; therefore it is him and not someone else who gets extra service.

This extra service may be due in the lifetime of the insured individual or may be due after the death of the insured person, so it may be two:

1. This extra service may be the development of a **mandatory periodical annuity increase**, what is due in the life of the annuitant. In this case the basic annuity must be set so, that it reaches a certain increase (e.g.: 5% beyond indexation) at certain regular periods of time, e.g.: in every 5 years,



if the mortality (the yield respectively) does not change into worse. If yes, this increase serves at balancing the worse mortality (yield respectively). The advantage of the solution is that the entire capital of the insured is used for what it was meant to, that is for the annuity, it means that it diminishes continuously, while it may easily happen that the bad years – for what this buffer was generated – occurs sometime at a later phase of the maturity. This is a strong argument in favor that the extra services would start the latest possible that is after the death of the insured person.

2. The service after the death of the insured person may be twofold as well: annuity or a lump sum service. In fact the first solution is a mandatory, posterior guarantee time, the other one is a type of supplementary whole life insurance. It is true for both solutions that advantages and disadvantages compared to the service due to the insured person during his life are the exact reflection of each other. The advantage of the service due after the death is that the service is rendered at the very end of the duration, when we can already be certain that the premium in the case of the insured was calculated in an adequate manner, the remaining provision of the extra service may be well used. An additional advantage is that during the period, if the buffer is emptied, it may be completely topped-up several times in theory, if the yields develop accordingly. However the disadvantage is, that it is not the insured person who enjoys this extra service (except if it is used for the funeral of the deceased – though this exception is relative). A further, though technical advantage of the bereavement service is that the environment of the insured person is interested in reporting the death of the insured, by this the insurer avoids paying the annuity after the death. Let us examine these two solutions in detail:

2.1. Mandatory, 1-2 year posterior guarantee time: In this case every annuity mandatorily must be equipped with 1 or 2 years end guarantee time (in the case of posterior guarantee time annuities additionally). If the mortality projection proves correct and there is no negative yield, the service of the guarantee time is transferred to the client (respectively to the beneficial person assigned by him). If not, this extra



service may be used to cover the loss that is the result of the incorrect projection – in this case the duration of the guarantee time decreases possibly to zero. The advantage of the solution is that the nature of the service after the death is identical with the one before the death; the disadvantage though is that it is hard to find arguments to extend the payment of the buffer, especially if the aim is to use it for the funeral of the deceased. Therefore the other solution is more logical in this case, which is

2.2. Mandatory supplementary whole life service that is lump-sum payment. If this whole life service is not defined as a fixed lump-sum, or is not defined in relation to the last annuity amount, but in a certain proportion of the provision. This helps the buffer to fit into the environment, where it is topped-up from the investment and mortality profit what is divided in proportion with the provision. The thumb rule for the size of the buffer can be what longevity increase it must compensate. For example 5% of the provision as buffer can compensate roughly 1 year longevity increase (respectively more by topping-up, with “good distribution” in time).¹⁸ The problem of non technical nature concerning the solution is that it is not certain, what every insured person has a heir, or is able to assign a beneficiary, though it may be

¹⁸ The buffer so defined may be calculated as follows:

If we use 0% technical interest rate, then the premium and the provision of the annuity insurance can be calculated with the help of the remaining life (the regularly marked at the age of x as: e_x). Here the insured at the age of x may receive the following monthly (primary) annuity from a start-up assets of „Ko” size (for the time being eventual factors of differentiation are disregarded):

$$\frac{K_0}{[12 \cdot e_x + 1] \cdot (1 + \lambda)} \text{ where } [12 \cdot e_x + 1] \text{ is } 12 \cdot e_x + 1 \text{ of the total, } \lambda \text{ means the}$$

expense part (in proportion of the net premium).

With the incorporation of the buffer this formula changes for the following:

$$\frac{K_0}{[12 \cdot e_x + 1] \cdot (1 + \lambda + p)}, \text{ where } p \text{ is buffer defined in proportion to the provision,}$$

where not partial expense is accounted for the buffer, what wouldn't be justified either.

In the case of technical interest rate different from 0% similar ideas may be created but a bit more complicated way, but the following statements are still valid. Though there is a good chance, that we can not top-up the buffer, respectively it may happen in a few cases.



bridged if it is used as a default for the expenses of the funeral (or for a part of it).

In summary, if we use buffer, then most of the arguments support solution 2.2. However this may be applied in a somewhat modified form:

1. The buffer is not necessarily used for the extra service after the death of the given insured, but in theory the buffer of the deceased people may be distributed among the insured still alive. The advantage of the solution is that by this the entire assets are used for annuity service, the disadvantage is that in respect of the insured person it is an even less fair solution.
2. The buffer may be applicable to a certain extent for smoothing the yield fluctuations, though this requires a somewhat more complicated operational mechanism than the above one. In this case the buffer is not only topped-up (if earlier some of it was used), but it is topped-up to a certain level for slippage (e.g.: up to the twofold of the original value). This may happen if as a result of the combined mortality and investment outcome the annuity would increase above a certain level (e.g.: the inflation of the previous year) and the buffer would not yet reach its maximum value. In this case one may say that a part of the extreme profit is used for a provision for worse years. The buffer could be used if it would be in an over topped-up status (that is beyond the initial, normal value), and the annuity increase would be below a certain level (e.g.: half of the inflation of the previous year) – maximum up to the inflation, respectively to the normal value of the buffer. This is a non-biased method for smoothing the fluctuations, so if it is important to operate such a smoothing mechanism due to important (political) considerations, it should be made with a more or less objective method. The disadvantage of the solution, that it takes potentially much higher provision from the service of a given annuitant, than if the buffer would not be used for smoothing.

I have not found international practice for using buffer, so as far as I know, this is my own proposal.



2.3.3.3.2. Collective provision

Interestingly, according to my experience experts of pension funds do not intend to apply the above individual provision for smoothing, but a collective provision separate from the provision of the individual insured persons, a “demographic and/or return equalizing” provision, despite of the fact that compared to the individual provision it has serious fault. The essence, (similarly to the individual provision) is that at the very beginning when the insurance policy is taken, the provision put aside from a part of the premium is used should the annuity decrease, and it is made up at the very first time, when it can be done without any decrease in the annuity. The making up is done from the return (and from the eventual mortality profit), until a certain predefined level (so even during several years, when as a result of this making up the annuity does not increase) as it happens in the case of the above individual provision. Though the collective provision entirely belongs to the risk community, it is not denominated – this is the difference compared to the one generated in the individual provision. Therefore a serious problem of equity emerges in respect of this provision, especially if the provisions get collected for a too lengthy period of time, respectively they are not used for a too long time, since this would be primarily used in favor of such insured people who did not contribute, or did not contribute in an adequate extent to the generation of this reserve (since a part of those people died in the meantime). In the case of this solution the client loses the right for his own money, therefore the problem is better managed by the similarly but by the correctly operating method of individual provision – if the idea emerges at all, that the mortality loss is delegated to the insured individual, because there is nobody else to bear it.

2.3.3.4. The solvency capital

The use of solvency capital differs from the reserve-type solutions above, that the financial consequences of volatility has to bear directly by the provider, not by the insured. Naturally, here also important the word “directly”, because the potential use of solvency capital will be calculated by the provider into the fee of its service, so it will be paid eventually by the insured.

In case of annuities this kind of use of solvency capital is really its most important function. However it is useful first to review why the law in general is used to



“subscribe” the use of solvency capital in case of assurances. This review will help us to understand under which circumstances can appear at all the use of solvency capital instead of use of reserves.

2.3.3.4.1. The function of solvency capital in general

The solvency capital enables the insurer to perform its undertaken services for the client with great probability, even though obstacles would occur. These obstacles may be of the following type:

- **Insurance technical risk** – the payments made by the insurer becomes higher than calculated either due to transitory risk fluctuation, or to its long term risk increase. In the case of annuities it means that as the result of mortality developments the annuity would decrease from one year to the other and the excess yield, or the provision generated for this would not be sufficiently balancing that (if the regulation allows at all).
- **Investment risk** – the insurer does not attain the yield promised to the client. This means, in the case of life insurances (including the annuity insurances), the yield adequate to the guaranteed technical interest rate and eventually to the guaranteed increase of service (if it exists). The risk means the risk of smaller yield than the technical interest rate and guaranteed increase of service. At a 0% technical interest rate, without guaranteed increase of service this means the risk of a negative yield – if this is not balanced by positive mortality outcome (provided the regulation manages these two together).
- **Risk due to the options and guarantees** undertaken. To a certain extent the above two risks were like that, but it can be imagined, that there are other guarantees too. For example this may be a repurchase (what generally does not exist at life annuities). In the case of life annuities it is generally difficult to identify such risk, but to a certain extent the risk that the client transfers his provision to an other provider (if the regulation opts for an annuity model containing this possibility, may be something like this. Technically this is a risk of repurchase, and the problem is that a portion of the provision must be liquidated, what causes a loss.



- **Operational risk** that is some problem occurs in the operation of the insurer (either natural disaster, human intervention, the database of the insurer is damaged by theft, the operation turns to be difficult or impossible, resources must be channeled to unforeseen places). This is a risk present at any kind of financial service provider.

I have to separately mention, that the first mentioned risk of insurance technical nature – that is the size of fluctuation in payments gradually decreases, so the smaller the stock the fluctuation is the higher. Therefore the regulation requires a solvency capital generally in proportion to the size of the stock, but it sets an absolute minimum, so that the fluctuation of small stocks can also be managed. I will elaborate on this topic later.

2.3.3.4.2. The correlation of the solvency capital and the ownership

The funds are special institutions among the Hungarian financial service providers. They are specific, because – contrary to any other provider (and to the common sense) only they do not have capital, not even solvency capital. Looking at the above functions of the solvency capital, it is obvious that in the phase of asset collection (this is the phase, when all the private pension funds and most of the voluntary pension funds are at the moment) the single operational risk may occur from among the above risks, therefore in this phase the lack of capital is much less a problem¹⁹ than at any other financial service provider. Nevertheless all these change in the phase of annuity payment, all the above risks may manifest then. The actual regime (the lack of solvency capital may be regarded as such) of the Hungarian funds are exclusively and solely designed for asset collection phase without any respect to the phase of annuity payment; perhaps at the time of elaboration of the regulation the 15 years, after which the annuity payment must start appeared to be incalculably far.

However unusual is the lack of solvency capital at the funds, we have to say, that it is entirely in harmony with the ownership conditions of the funds, namely there are no actual owners only quasi owners exist. The owners of the funds formally are the actual members, but equal rights are for those who contributed by a hundred million



Hungarian Forint to the establishment of the fund as those who paid the first thousand Forint yesterday. In such a situation nay capital accumulation would be unfair, since the person from who it generates can not expect any yield neither more voting rights²⁰ (in practice the operation require equipment, computer, etc., so there is some sort of capital accumulation). If the funds had capital, that would be accumulated from the contribution of the starting generation, and the next generation possibly would not contribute to that, but had the funds been eliminated the capital would be distributed among members of the next generation what they did not accumulate. This would be unfair and deeply problematic, the actual system e.g.: the lack of formal capital is much more correct solution. In summary the actual ownership structure and the regime of the solvency capital is in good logical correlation with each other.²¹ In the capital accumulation phase the actual system works satisfactory – with strong supervision. However it is not sure that this logically coherent system is able to manage every problem, the problem of annuity only with some restrictions.

Solvency capital is needed if we want to avoid that the client (member) bears certain risks, primarily the risk of eventual bad investment outcome and serial mortality losses what may lead to a decreasing annuity. Based on the above, a financial institute having quasi owners can not even theoretically have solvency capital. If funds are allowed to provide annuity, we have to take into account, that all the risks are directly born by the annuitants that can be diminished by the above buffer-technique.

This specifically sharply appears when the annuity starts when the annuitant risk community is small, so the mortality fluctuation is necessarily big. The solvency capital has specifically big role at that time (see the management of small stock).

¹⁹ It seems that in Hungary detailed regulations and supervision bridges over the lack of solvency capital for the coverage of operational risks. On system level these regulations are much tighter than the ones applicable on other financial service providers.

²⁰ At least formally, legally. Actually the majority of the funds de facto, but not de jure, are multinational financial service providers, who has a strong voice in how the fund operates, but the reason is more organization sociological and not legal one.

²¹ Out of curiosity let me note that from this aspect the Hungarian regulation is inconsistent, since the regulation of institutions totally similar to the pension funds is nota s fair as this, therefore it is deeply problematic. At insurance associations the solvency capital is required, but i t has no defined owner (neither has the associations itself). Nevertheless this inconsistency would be difficult to correct by cancelling the solvency capital requirement at associations, because insurance risk could not be undertaken at all without solvency capital. It may be remedied only by the elimination of the association form (in case of insurers).



In reaction to the above arguments it would be highly problematic if the regulator ordered that funds may provide annuity but to do so they must accumulate solvency capital from the contributions of the members. On the one hand it would be problematic due to the above explained reasons of equity, on the other hand because this would not solve the problem. The risk fluctuations are the highest at the start-up of annuity payment, this is when the solvency capital must be available, and so there is no time for a slow accumulation in this period.

If funds started to accumulate the solvency capital in the phase of capital accumulation for the phase of annuity payment would also be problematic, namely the free option of fund does not mean that somebody can take the assets accumulated by his contribution to an other fund. Beyond that there is another essential problem.

A very important and deeper connection between the solvency capital and the ownership is that the solvency capital must be always available in its totality that is even at a time, when its major part was use for the purpose what the solvency capital stands for. In such a case the owner must immediately top-up the solvency capital to the adequate level, and it is done²², because there are owners. At funds without a real owner it may happen that the solvency capital is topped-up once in a long time to an adequate level, but if it is used once, the fund will be in the lack of solvency capital for a long time, what endangers the provision of annuities. In the lack of owners nobody is interested in putting the missing solvency capital into the fund, since this is not coupled with any advantage. If the existing members, as owners were forced to do so, for them a better option would be to exit the fund and join a fund which is not in the lack of solvency capital where this problem is not theirs. The only option in such a case is that the supervisory authority prohibits the further operation of an insurer that has no solvency capital, so if the regulator required solvency capital without owners it would program the system to face frequent bankruptcies with high probability.

Therefore it can be stated, that the solvency capital is only characteristic at the profit oriented financial institutions (like insurance companies), so it is only them who can solve problems concerning solvency capital and not the funds. Funds should either

²² Should they fail to do so, the consequence is a supervisory intervention in this framework the provider is possibly liquidated and clientele is transferred to an other provider. The solvency capital allows for the seamless management of the portfolio transfer without any damage to the clientele.



not be allowed to provide line annuity or they must be transformed into organizations with actual owners the other possibility is to recognize that risks in respect of annuities are directly born by the annuitants (in practice it means that in case of unfavorable mortality outcome the annuity may even nominally decrease from one year to the other – with the use of buffer it may only happen in the case of consecutive bad years).

2.3.3.4.3. Solvency capital in the case of the annuities

Based on the above solvency capital in respect of annuities may and must be required from insurance companies, respectively from providers operating eventually in corporate form, and only in the case of annuities provided by such providers may be required that the annuity does not decrease from one year to the other and /or in the case of bad mortality outcome. The actual European and with this Hungarian regulation on insurers the requirement of solvency capital is already included, however one may recognize that it does not take into account the specific risk, therefore it is worthy for fine tuning and revisiting in respect of annuities. If the regulations permits the insurers and funds to provide annuity this requires the operation of two different systems from solvency capital point of view. (Here the general question whether two different systems may be operated parallel, and whether the clients may have a transparent view about it can be raised.)

In the case of annuity the solvency capital gives protection to the insured on the one hand against the mortality loss, or in the case of using a buffer against a sort of overspill of the accumulated effect of consecutive unfavorable mortality – and in this context – in the event of extreme investment outcome. Beyond this in the case of small stock the solvency capital must be available if the collected premium - due to different composition of insured from the originally assumed one - is not satisfactory for the initial provision. All these are regarded as a kind of option, what is purchased by the insured from the insurer, and for what the insured must pay an option premium. This option premium may be partly inbuilt into the premium of the annuity or into the provision yield of the annuity. From this regard mainly the technical interest rate may be a kind of regulatory parameter. The higher the technical interest rate that is the annual yield expectation, the higher the probability that this can not be performed in a given year, so the higher the probability for the call for the option, that



means the option itself if the more expensive and vice versa. This is an argument for having a mandatory technical interest rate as small as possible, 0% ²³, what means at the same time, that the option premium is restricted to the minimum, since with an interest rate of that small there is a minimum of risk that the yield of the investment does not reach that. ²⁴

In the case of annuities the necessary magnitude of the annuities depends on several factors, therefore the legal regulation must distinguish along numerous factors. The major factors influencing the solvency capital requirement:

- Smaller solvency capital is needed for annuities with guarantee time, than for annuities not including guarantee time, since the front end guarantee time decreases the risk of the insurer and the use of back end guarantee time is nothing else than the sale of two insurances with opposite insurances (life annuity and whole life) which partly annul each other,
- Within annuities of front end guarantee time, a minimum of solvency capital is sufficient during the guarantee time (since there is no insurance risk there), should the regulation allow that solvency capital is provided by somebody else than the other part of the annuity,
- If the annuity may be split in time for different providers, then a smaller solvency capital is required for the temporary annuity paid at the beginning of the period, while for the parts of the annuity that last until the end of the life bigger solvency capital is required,
- The solvency capital requirement is increased if the premium according to the tariff administratively maximized,

²³ Naturally the technical interest rate may in theory be negative too, so 0% is not the smallest possible value and for negative interest rate no „natural” low limit can be set, not even 100%, since for shorter period an interest rate below 100% can be imagined, though the annuity is fundamentally set for a longer period than one year. However the negative interest rate must be looked at as such a curiosity, for the application of which a very strong, specific reason is needed, therefore I maintain that 0% is the possible smallest interest rate, even if this is more a physiological than theoretical limit.

²⁴ This is very much strongly dependent on the investment strategy what in relation with the interest rate. The lower the interest rate, the investment strategy may be more daring, or may be such that brings higher yield on longer term, but the volatility of the yield increases. So in certain sense the 0% does not decreases but somewhat increases the probability of a negative yield.



- In the case of unisex annuity requirement smaller solvency capital is satisfactory for annuity due for two lives than for one life, since the uncertainty as a result of unisex annuity requirement ceases to exist.

2.3.3.5. Separation of the normal and the peak risk – common treatment of the peak risk

In respect of annuities the extended longevity (longevity risk) may be regarded as the peak risk for the insurer. Primarily the starting-up institutions, that provide lifelong annuities are exposed to this, annuity providers that assure deferred annuity (the longer the deferral) the less²⁵, temporary annuity providers are very little and providers of annuity certain are not exposed at all. As the immediately starting lifelong annuity may be split into temporary and deferred annuity amount, by this method the normal and the peak risk of the annuity can be separated while the peak risk may be diminished. So practically: an upper age limit may be determined (e.g.: 80 or 85 years) and the immediately starting life annuity may be split into these two parts. As in our days it is difficult to imagine annuity of unchanged amount, that is non-indexed annuity, therefore a special care should be taken that the service provided in these two different phases not “slip apart” as a result of the different indexation. There are two ways of protection against “slipping apart”:

1. The provider, who gives the service in the first phase, manages the provision of both parts in the first phase, and invests them in an identical way, as long as the client reaches the age limit. This way the indexation of the two parts remains parallel, and the annuity amount turns smoothly into the second phase, and the provision for the deferred annuity is transferred to the provider undertaking the peak risk only at the beginning of the second phase.
2. The provision of the deferred annuity is transferred already at the very beginning to the other provider; nevertheless investment may be made exclusively into fixed yield bonds (indexed to inflation) at both places, so the indexation of the provisions remains parallel.

²⁵ This is only true if we compare the deferred annuity with that spontaneously starting annuity where the entry age of the insured persons is identical. But if we compare those two cases for example, that somebody enters into an immediately starting annuity at the age of 62 or enters into a 12 years deferred annuity at the age of 50, then it is not clear the longevity risk in which annuity is the higher.



The deferred annuity provider may be of three types:

1. the same provider who provides the annuity of the temporary part. The provider may do this, if the solvency capital is generated for the entire life annuity that is for the peak risks as well.
2. a second provider with bigger financial muscle than the first one, is a kind of annuity reinsurer,
3. a central provider established or selected by the state (in fact an “official” reinsurer).

In item of 2. and 3. the need for solvency capital is smaller at the temporary annuity provider than if he held the entire risk. What is more, if the temporary annuity is annuity certain, then this service may be provided with having a minimum of solvency capital, in fact the actual Hungarian funds are adequate for this.

The longevity risk in item 1. is undertaken by the original provider (who is enabled for that by the solvency capital of adequate magnitude) but in item 2. and 3. it is undertaken by the reinsurer, respectively in item 3. finally it is the state. The state is encouraged by this that the mortality projections prepared by it are as accurate as possible, because this enables the state to avoid a great burden of the peak risk.²⁶ (If the projection is correct, there is no extra burden.) With an adequately selected upper age limit it is achievable that the central provider belongs to the range of smaller providers (in respect of provision), so the major part of the annuity risk further remains with the market and is not transferred to the state. However due to the upper limit of the service the temporary annuity means a more calculable situation, respectively it is easier for the market players to calculate the length of time they have to bear the extra burden if they made a miscalculation.

2.3.4. Mortality projection and managing the longevity risk in the technical literature

According to Mehr-Gustavson [1987] (page 527-528) different mortality tables are used already early for annuitants in America, and since 1949 the improvement in the mortality tables have been taken into account. It means that the projection of mortality in the annuity calculation started at that time.



According to an English textbook, what is also translated into Hungarian (Hylands-Gray [1992]) at the calculation of mortality the future improvement must be prudently assessed. The textbook also tells, that examination of the annuitants' mortality was conducted in 1967-70 what gives the bases for the applied standard mortality tables. These were not adequate for the premium calculation, only for comparison because the future improvement of the mortality was not taken into account. Therefore an adjustment was applied as they started from the table of 1968 and afterwards they deducted $1/120$ from the age in each year (in the case of the relevant generation), and so they calculated the premium. Beyond this when it comes to pricing they take into account the own annuitant mortality experience of the given insurance company. According to Hylands-Gray [1992] the projections retrospectively reflect a mixed image. Based on the examinations of 1975-1982 the predicted male mortality was near the actual one but in the case of women it was highly underestimated.

One of the major problems for the annuity providers is the underestimation of mortality improvement according to Blake [1999]. This is enforced with a reference made to FSA 2002 statement by Davis [2003], according to whom the major risk is that the providers underestimate the longevity of the annuitants.

Trying to give advice to the insurers Winkler-Mattar [1999] can see two ways of managing the risk due to the mortality improvement: 1. the insurers give option for transformation into annuity for the client but no guarantee is given for the price but they rather let the client leave for an other provider, 2. the use of mortality tables of generations (I suppose that this means the projected mortality tables according to cohorts!).

Booth-Chadburn-Cooper-Haberman-James [1999] (page 226-7) calls the attention for the increasing longevity in the developed countries, so the annuity calculations must take into account the anticipated future improvement of mortality (but it does not give details of how this must be made!). This is done by Antolin [2007] according to whom mainly stochastic methods shall be used in the mortality

²⁶ According to some experts it is worrisome that the state influences several important parameters at the same time. Therefore it is possible that the mortality projections and the central provider, if there would be any at all, are strictly separated from each other.



projection, the bases of which is laid down by Lee and Carter in 1992 (see Lee-Carter [1992]).

Winkler-Mattar [1992] presents the method the projections was conducted before Lee-Carter, according to whom with respect to the experienced mortality decrease q_x must be diminished at the projections. It is not reasonable though to make it linearly, because the probability would sooner or later become negative. Therefore the formula is widely used $q(x, t) = q_x \cdot e^{-\lambda_x(t-t_0)}$ where λ_x is the age specific annual improvement trend and t_0 is the year for which q_x is originally valid. This is a simple model, however according to Winkler-Mattar there is no empirical proof for its correctness. Therefore other methods are also applied. E.G.: the “frailty” mode. Where the population is split into stronger and weaker sub-populations with the presumption, that the evolution of medical science has a higher impact on the weaker population. Though it seems, that the total disappearance of different illnesses would not radically lengthen the longevity, what is more, it has never happened as a result of medical science development, the increase is rather caused by other factors.

Whatever method was applied for the projection those did not reflect the actual trends (as it is stated by OECD [2008g] too). Barnshaw-Lster-Steinmann [2008] page 11) quotes Chris Shaw’s analysis (Fifty Years of United Kingdom National Population Projection – Shaw [2007]), according to it a boy born in 2010 was predicted to live 71 years in 1997, in 2000 it was already 77 years. The consequence is that there is a great deal of uncertainty in the projection (in other words the uncertainty remained even two decades after Hylands-Gray in respect of projections). Retrospectively it can only be stated that according to the 2007 OECD data the longevity at birth and at the age of 65 increased between 0,6 and 2,6 as an average in every 10 years between 1960 and 2000 in the OECD countries. (OECD [2008g])

We may say that in the case of annuities the mortality projection is obvious in the international technical literature. Despite of that it does not even appear in the Hungarian technical literature, at least not in respect of annuities. In respect of private pension fund annuities Michaletzky [1999] writes the following (page 106) “...it would be important to define precisely mortality tables of which date are allowed to/ must be used at the evaluation of services. At the determination of annuity provision the actual tables (by gender) or the table of the actual year.” This may be interpreted, that



there is not even a hint about the need to use the projected mortality tables at the annuitants. Arató [2006a] only states that "...20-30% is the inaccuracy in the determination of the remaining longevity" but he does not mention the importance of the projection (though it may be traced in his text).

No wonder that the phenomenon making projection necessary is only mentioned (quite detailed) in the international, but not in the Hungarian technical literature. This is the lengthening of the lifetime the related risk respectively (longevity).

The expression (since I speak about the technical literature in English language, I use the English version of the expression) made a quick career. In his book Mehr-Gustavson [1987] the word longevity does not appear in the index. In the next decade Black-Skipper [1994] already uses the word longevity on pages 427-428, but only in the context that due to the extended lifetime the pension scheme is important for workers.

In 1999 Winkler-Mattar [1999] already demonstrate the phenomenon with a lot of tables from Swiss Re, respectively the increase of the life expectancy at birth at the age of 65. The reasons: better health care, changes in nutrition habits and other changes in the life style (e.g. decrease in smoking). According to the Harvard Medical School those women who gave birth in their 40-ies live longer. So the general extension of giving birth contributed to the extension of the lifetime. The self-employed also live longer than the others. Nevertheless he says in relative terms, because he thinks that in the case of a heterogeneous portfolio the mortality caused extremes as a result of mortality are higher than the ones of longevity, so the longevity risk may not be separated in the case of such portfolio. His publication dealing with the Swiss Re (Scotti-Effenberger [2007] has a title in the first sub-chapter "Demography challenge and longevity risk". According to this, the life expectancy at birth at the second half of the 20th century increased as an average by 4.5 month. This may be primarily but not only due to the decrease in baby mortality, and it is even less the case in our days. The consequence of all these trends is as follows according to him:

- decrease in the state funded pension and the system shifts from the DB to the DC
- increased mobility breaking away the previous family ties and therefore the young ones tend to take less care of the old ones



- in summary we may only count on ourselves!

According to Antolin [2008a], respectively OECD [2008a] the pension systems are mainly affected by uncertainty of the extended lifetime, respectively two important risks shall be managed: the risk of timing of transformation into annuity and the longevity.

This management of longevity risk, respectively the exploration of it is very often discussed in the English literature. In 1999 Blake (Blake [1999]) spoke only about the 'natural' coverage of the annuity longevity risk, that had been long discussed in the literature, that is he proposed that the provider sells not only annuity but life insurances of the opposite risk, so the risk would partly balance each other. The same notion is mentioned by Davis [2003] as well. (We have to add, that in two years time Blake expressed a completely different idea about the longevity coverage, what have since become very popular in the literature (Blake – Burrows [2001]. Winkler-Mattar [1999] mentions too, that the whole life insurance and the annuity compensate each other's risk, and he notes that here there is much less possibility than the conventional wisdom states, because this would only be entirely true if the same person concluded the two insurance contracts. However according to the experience the clientele of the two insurances is different even at the same provider. Beyond this Winkler-Mattar [1999] mentions that insurers supposed during many years that the increase in longevity is over compensated the investment yield (though e.g.: in Germany the regulation restricted the direct transfer of the yield for this purpose). This also may be interpreted, that my proposal, according to which the mortality and investment yield are to be treated together has been a well known and wide-spread practice for long. According to Winkler-Mattar calculations [1999] e.g.: in Switzerland in the case of entry at the age of 60 and 75 there was a need for longevity compensation as an average between 0.31 and 0.43%, in other words according to him the mortality risk is significant compared to potential investment profit to compensate that.

From an other angle Wadsworth [2002] sets a radically different proposal about it (from an aspect he gives a new definition of risk management at the expense of the excess yield), according to him the longevity risk, or at least a part of it shall be put on



the client via the less risk undertaken by the providers. According to him this risk is not even priced correctly in our days.

He also prepared a report commissioned by the Association of British Insurers – ‘ABI’ in September 2003, “The Future of the Pension Annuity Market” Wadsworth [2005], in this he determines that the longevity risk has a very restricted secondary market and there is a restricted chance for the development²⁷ of a wholesale market for that. He also finds important the supply of long term bonds generally in respect of annuities, and he can see problems and reasons for action in this regard.

According to Scotti-Effenberger [2007] there is an extremely high, mainly ignored risk of systematic longevity increase in relation with annuities in the entire population. According to them the longevity phenomenon is well understood from demographic and macroeconomic point of view, but it was not deeply examined from insurer’s risk management aspect – this is what they do. Since this is a systemic risk, therefore it can not be diversified, that is the traditional techniques of risk management (e.g.: reinsurance, stock unification) do not work. In their opinion governments are the best to help risk management; they have a major role in that. Their main tasks: awareness rising, issuing the necessary instruments needed to cover longevity risk, diminishes adverse selection and encouraging self-care by tax incentives. The governments may take over longevity risk from the providers, because they are anyhow exposed to that because of the Pillar I of the pension system. Parallel with the governments this risk might be traded in the financial markets as well, where mortality indexes may be used for benchmarking. So far very little such securitization took place, but the Swill Re already issued a few of this. The market methods for the coverage may be: interest rate swap, participation in businesses having opposite mortality dynamics, investing into businesses having different mortality dynamics, buying longevity bonds and the synthetic proxy hedge. These latter two are still in experimental phase; there are no sufficient available instruments like those yet. Mainly pharmaceutical companies, old-age homes, biotechnology companies win by the extension of longevity. Investing into these may be regarded as natural hedge. Though the problem with longevity bonds is

²⁷ It appears that the development for such a market happened in the United Kingdom in 2010. IPE reported in January 31, 2010, that 8 banks, insurer and reinsurer established a Life and Longevity



that longevity risk is replaced by credit risk, what is not necessarily desirable (except if the state issues such bonds). All in all Scotti-Effenberger [2007] insurers must do a lot for better pricing of annuities, and for minimizing administrative and trading expenses, as well as to clear the basic product form the unnecessary frippery. They can see much more to be done by the state:

- to issue better mortality tables by this enforcing transparency in longevity
- to raise awareness of the extended longevity (according to a UK survey people tend to underestimate their lifetime by about 5 years. According to an other survey if a longer life is expected they are more inclined to conclude private pension insurance)
- to make information available about the financial situation of the retired people - see the Swedish "Personalregister".
- To strengthen the financial training for citizens
- To issue financial instruments for the coverage – unfortunately only a few countries have 30 year government bonds, 50 year does not exist
- To increase the volume of business by making annuity mandatory and to decrease auto selection
- To develop and establish the framework for supervision
- Harmonized and motivating taxation – in EET system

According to Barnshaw-Laster-Steinmann [2008] the longevity risk, e.g.: the uncertainty of how long our life would be has two components: an individual and an aggregate. The individual risk may be managed by organizing the risk into pool (what is nothing else than buying annuity), though the aggregate part can not be diversified and at the moment it is also difficult to cover. Managing annuity risks (in general in DB systems), may happen via the so called liability-driven investing (LDI), which means that assets are adjusted to liabilities so that investments are mainly made into long term bonds. (So LDI is the same strategy what is called ALM in insurance, an interesting question why this needed a rediscovery and a new name.) According to OECD [2008g] the ALM requirement have already been implemented at many places. So the similarity of assets and liabilities is a stronger requirement by now.) The LDI strategy

Markets Association (LLMA) to promote popularization of coverage for longevity risk and trading. (IPE



and the implementation of inflation-swaps and swaptions allow the adjustment of the term and the convexity without investing everything into bonds, in the lack of adequate long term bonds. In the case of derivatives the basis risk must be taken into account, as they do not precisely match the assets. From the point of view of an institution adequate risk management may be the white labeling, when the product of a professional insurer, of the manufacturer is traded by another institution, distributor in its own name. Often this is jointly developed. Though the transfer of the longevity risk does not really work so far, unfortunately there is not as many longevity bond vendors as many potential buyer exist, though the market exists, but it is very much restricted and sluggish. The secondary market of life insurances, the settlements market, may be a solution what mainly works in the case of risk insurances, but it is little transparent and very much unique to be a mass solution. However the market of longevity risk is indicated by the list published by the Financial Times about the available longevity risks.

Antolin [2008a], respectively the basically identical OECD [2008a] speak about the difficulties of the market coverage for longevity risk, since according to him actually the longevity swap options ceased to exist, respectively the instruments to cover longevity are absent. The experiments of private market failed. The projected EIB bond issue failed too in 2004 (see OECD [2008g], and it would have been anyhow too expensive). Antolin [2008a], respectively OECD [2008a] finds the reasons that there is nobody who profited from the unforeseen raising of age limit, so there is not enough issuer (see OECD [2008g]. Moreover the issuer can not really have an exit from such bonds, what makes it even less attractive for the potential issuers. Therefore he suggests, similarly to Blake or Scotti-Effenberger that the government would issue such instruments. Nevertheless the argument against this is that the government has much enough longevity risk (so this is an obstacle contrary to Scotti-Effenberger [2007]. Due to the crisis of the swaps these are not really considered.

Antolin [2008a], respectively OECD [2008a] notes that not only longevity bonds, but the simple long term governments bonds are also missing. So it is the government who should issue such bonds, and as for the longevity risk management at least it may

[2010] respectively The Economist [2010]



stimulate the market by publishing longevity index. (It is repeated by OECD [2008h] and OECD [2008g] too. These are all necessary for the market to voluntarily sell annuity products (Antolin calls them decumulation products), since the major obstacle is that the stakeholders think they can not cover the risk.

2.4. THE ISSUE OF MANAGING A SMALL STOCK

The above annuity formula implicitly supposed that the stock of insured is adequately large, since mortality according to the original presumptions is possible only then. Namely any kind of mortality or survival probability can only be generated as the quotients of integers (the ratio of deceased in relation to the people at the beginning of the period) only in the case of adequately large number integers (stock).

If the mortality result forms part of indexation, it adjusts the provision via the indexation mechanism. However in the case of a small stock it may appear, that no insured people die in a certain year, what may cause a very big mortality loss, that may even surpass the entire investment extra yield; though if at least one insured dies, then in that particular year very big mortality profit may arise (in proportion to the provision of the whole stock). Therefore there may be extreme fluctuation in the annuity of insured belonging to a small stock from one year to the other. There are two fundamental methods of protection against these fluctuations:

1. By a pool, that is by the unification of stocks. In such a case the stocks of different providers are unified into one pool, so these are organized into a bigger pool, where these fluctuations would necessarily become smaller
2. By solvency capital.

If the regulation allows that funds also provide life annuity, only the first option is available for them (it is worthy to render it mandatory with no respect to the magnitude of the stock). However at the launching of the annuity system the entire risk community is small, so the problem of a small stock would almost necessarily emerge even if there is a pool, so the pension fund providers may even need budgetary aid. While the pool is almost unavoidable for pension fund providers, it may also be useful for providers (insurers) having own capital, so it is useful if the regulator allows the formation of pools on voluntary basis. What is more, it may even be made mandatory, so the pool may work as a supplementary risk mechanism, that adjusts the



haphazard compositions of the risk community at the different providers, what they can not manage with their natural tools, as differentiation (e.g.: according to gender) is banned. Naturally it may happen, mainly at the beginning that the stock, what is organized into a pool is not sufficiently large (in the case of providers having own capital), therefore the implementation of the second solution may happen, parallel with the first one.

Services provided against the solvency capital may be regarded as an option assured to the insured people, the price (premium) of which is paid by the client either as the entrepreneurial fee or as extra yield of the insurer. In the case of a large stock, and if the mortality result technique was taken into account at the indexation is implemented, especially if the technical interest rate is 0% there is a little likelihood of using the solvency capital, but if the stock is small there is a great chance of using the solvency capital, due to the extremely fluctuating mortality result. The differentiation in a small stock would further split it into even smaller fragments, if it happens in the case of provisioning.

If according to the regulation even if the mortality result is treated as part of the indexation in normal cases, the providers have own capital, in the case of small stock (presumably at starting-up) this indexation rule would be reasonable to suspend, based on the above reasons, and instead the following approach should be implemented (independently whether it is the stock of a given provider or it is a pool):

1. as long as the entire pool reaches a certain threshold the entire mortality result is of the provider (or in the case of a pool: of the providers), that is not made part of the indexation. In this case indexation takes place exclusively on the bases of the investment result. The transitory mortality losses are financed from the solvency capital by the provider(s), the mortality profit will be one of the sources of topping up the solvency capital for them. As long as this takes place the provision calculation is made by differentiation according to risk groups, but the mortality result of the different risk groups are obviously not treated separately.
2. if the entire stock reaches the threshold, but there are such risk sub-groups that don't, in that given year the risk groups shall be consolidated starting from the group of smallest number until the number of people in the



consolidated group reaches the threshold. The mortality result is accounted for in the consolidated group exactly as below, but within that not by risk groups.

3. if every risk sub-group reaches the threshold, the mortality result becomes part of the indexation and the mortality result is separately treated for every risk group, the indexation of that particular group is made separately from the rest of the groups.

This initial use of the solvency capital may be looked at as the initial investment of for profit, capitalized provider, the return of which is the option premium collected for the use of solvency capital of a future larger stock.

Calculation is needed to determine the above mentioned threshold. The principle of the calculation may be that the stock shall be as large as the probable fluctuation of the mortality not exceeds a yield level that is sustainable on longer term.

2.5. THE PROBLEM OF HOMOGENEITY OF THE STOCK OF ANNUITANTS

Homogeneity from risk point of view of the annuitant stock may be examined from several aspects: e.g.: homogeneity by gender (or the insured are only women / men), by year of birth (the insured people belong to identical cohort), etc. These are very exciting problems, but homogeneity by gender may possibly not be achieved (or can only be achieved by illegitimate means). Homogeneity by cohorts can not be achieved by differentiation (no legislator may expect that pension savings would be exchanged for annuity in 50 years time based on a tariff carved into stone today). So from among the possible problems of homogeneity I highlighted the issue of homogeneity by the magnitude of the annuity; below I examine this case.

2.5.1. The possible correlation between the magnitude of the annuity and the longevity

Above I implicitly assumed that the annuities are of equal magnitude by writing the premium and the provision formula of life annuities uniformly at 1 HUF. Naturally the actual annuities by size would probably indicate a relatively big dispersion; this would not cause any problem if we may suppose that there isn't any kind of correlation between the size of the annuity and the remaining life expectancy. If we differentiated the stock of insured by several different parameters (mainly: health



state, lifestyle), probably this would be the case in the different subgroups and naturally in the entire stock of insured). Beyond this it is also possible; that the annuity level would become homogeneous to a certain extent within the different sub-groups (even though it is also possible, that the average annuity of the different differentiated sub-groups would significantly differ from each other). In most countries, as in Hungary this many folded differentiation in the mandatory annuity is not possible, not because of technical considerations, it is possible that there would be a correlation between the amount of the annuity and the remaining life expectancy – as a result of not taking it into account.

For the time being there are not data for the correlation itself, neither for its rate, but in my personal view there is a strong likelihood of a big, positive correlation (so the bigger the annuity is the longer the insured lives). I hold this opinion despite of the fact that many experts has doubts on the basis that concealment of income is a general practice in Hungary, so in reality there will be many very rich people among those entitled for small annuity. As for my part I think that we could have clearer data if this phenomenon was not a matter of fact, put despite of this there can be positive correlation, since there are relatively less rich people than poor ones, so however the rich indicated among the poor ones distort the picture (mitigate the positive correlation), but would not change it. In addition this may change by the whitening of the economy (what may be significantly improved by a more equitable tax system what is forced on Hungary by the competitors).

In the lack of data it is mere speculation to claim the positive correlation, or the lack of it, nevertheless the problem is anyhow worthy to deal with, and calculations must be made on the basis of different available databases (see PKN database).

Beyond data collection and the digressive annuity table based on it there is an other possibility to access the problem; either this or the contrary is a priori eliminated, or at least significantly diminish its possible magnitude. This is homogenization indicated in the title.

2.5.2. Annuity homogenization

Homogenization means, that the possible difference between the magnitude of annuities becomes restricted. This means that the peak annuity is converted to



average annuity (in other terms: the annuities become capped). The justification of such a regulation can not be that this is clearly more beneficial actuarially, which results in the creation of a more calculable situation, which really means that a situation what is more beneficial actuarially can only be created as a consequence, if we find an other adequate justification for homogenization. Such a justification can be found if we look at the reason why the law makes citizens obliged to make pre-savings²⁸ for the coverage of their old-age financial needs. The state does not let people do this step without making it mandatory and it would as a maximum only give precise information about to what extent it would be reasonable to set aside from their actual income. The answer is that the time preference of people is generally not adequate, too many people prefer to consume in the present against consuming in the future therefore they are not inclined to save enough money on their own, which means that they would appear in the national social system as a mass of unsupplied old people causing an extra financial pressure. The state in order to avoid it obliges the citizens to make pre-savings. However the necessary pre-savings has an upper limit, beyond what the above problem does not show up, therefore beyond that limit there is no need for any obligation, the state can be satisfied with the common sense of citizens, in the lack of which old people earlier having good level of income may radically diminish his consumption but would not be reliant on the central redistribution, would not live on social benefit and would not suffer of privation. Forward thinking individuals should not be obliged by the state to make savings beyond a certain rate in such a system (private pension funds), beyond which they may have savings of better yield perhaps (e.g.: purchase and renting out real estate, own enterprise, etc.). There are serious arguments in favor of private pension fund savings, so of setting a cap on the mandatory contributions and by this the possible annuities may be made more homogeneous.

Homogenization may have different methods:

²⁸ It seems to be an enterprise what is not totally impossible. China as the proud country having one of the highest savings rate in world achieved such a situation so, that some decades ago they started a campaign by making saving a "patriotic obligation". (see: Akelof-Shiller [2009]) Obviously we can not predict that in a society what is much more individualistic than the Chinese one, like the Hungarian, what sort of campaign would lead to success, but even if it so happened the actual pension system could be switched over to this basis in some decades. In addition to this it is still not clear whether people obeyed or the campaign did not have an important role at all.



1. The system does not allow the generation of such annuities, and it sets a strong upper limit of the contribution payment (even by banning surplus money to be paid beyond arriving at a certain amount of assets). This is a better solution than the one (the actual Hungarian solution) that the maximum is always linked to the actual income, because it manages the problem if the income would drastically drop.
2. The regulation would set the same upper limit in (let's say the monthly) annuity, not for the contribution payment but in the exchange for annuity. The assets beyond the upper limit would be paid to the fund member in a lump sum. Compared to the previous solution this is better because it manages the problem of uncertainty in respect of the rate of capital exchange into annuities.
3. An other option is that the regulation would allow annuity to be beyond the upper limit but excludes the possibility of significantly big differences in annuities at different providers. This can be achieved so, that the capital of an insured person is distributed among so many providers as the services provided by different providers are smaller than a set limit (but only as many providers may be chosen as a maximum as many are necessary for this, in other words the annuity can not be intentionally fragmented). Though this solution is different from the previous two ones by splitting the higher longevity risk among the providers while it is still in the system.

Naturally homogenization naturally means the exclusion of not only the too high annuities but also the too low ones as well. One part of it is that the regulation does not permit the exchange of too small saving into annuity. Below I deal separately with this aspect. This is the issue of "absolute" annuity minimum.

The other aspect (the issue of "relative" annuity minimum) only comes into the picture if – among different annuity types – the client may opt for an annuity with guarantee time. If the guarantee time is possible, the guarantee has its price, what manifests in the diminishing of the monthly annuity that can be provided for the accumulated capital. Therefore it is justified to set such a restriction for the guarantee time, even if it is allowed, that beyond a certain (capital, respectively measured in annuity) limit the insured person may only buy annuity for one person without



guarantee time. From the other aspect it serves the homogenization of the risk community.

2.5.3. Homogenization in the technical literature

In the technical literature I have only found reference to homogenization at Winkler-Mattar [1999], who used it in the same sense as I do. He says that in the case of heterogeneous portfolio the mortality swings due to the heterogeneity are longer than the ones due to the long life, while longevity risk can not be evidenced at such portfolio. He also notes that heterogeneity is generally not diminished by reinsurance contrary to other life insurances. However he suggests as I do suggest solving problems resulting from heterogeneity, so he suggests to set a ceiling on the annuity payable, in general a particular insurer should restrict the annuity within its total business by an adequate bonus structure for example, respectively.

The Hungarian technical literature does not / neither use the expression homogeneity, though Michaletzky [1999] already proposes (with a reference to Stahl – not indicating the source!) that there might be a correlation between the mortality and the magnitude of the capital – he thinks to solve this by the preparation of an adequate mortality table.

2.6. INDEXATION

2.6.1. Possible forms of investing the annuity provision and annuity indexing

The traditional annuity-formula assumes that the insured person gets nominally the same annuity from the start until the end of life. From another angle it supposes that the yield on the annuity provision is precisely identical with the expected yield that is with the technical interest rate it would divert neither upwards nor downwards. In fact the annuity shall not be increased as a consequence of redistribution of yield above the technical interest rate (excess yield), nor it shall be decreased due to lower yield than the technical interest rate that is due to loss.

However it is obvious that the technical interest rate will be surpassed by the actual yield from time to time during the term, therefore the annuity will have to be indexed – especially in the case if the technical interest rate was set so that there is a great



probability for it to happen, and if the indexation is anyhow the expectation from a certain point of view.

In effect the possibilities (moreover the necessity) of indexation depends on the form how the provision is invested. The possible investment forms of the annuity provision are different from each other depending on who takes the investment risk. Naturally this has an impact on the annuity facility itself, and primarily on the indexation. The possibilities are:

1. both the provider and the client is exempt from the investment risk – it depends on the yield of the assets the provision was invested into. It depends on the inflation most frequently. The basis of the investment strategy is, that in such cases the provider invests exclusively into (good quality) bonds in such an expiry structure what is the expiry structure of the annuity portfolio. So the provider is exempt from the market fluctuations, since it invests only and exclusively into bonds kept until the date of expiry, what needs no revaluation with the changes of the interest rates. As long as only fixed yield bonds existed this strategy also meant that the annuity dues remained unchanged until the expiry, possibly (rarely) grew at a pre-determined rate. In our days bonds with yields depending on the inflation are wide spread, so with this strategy the annuity indexed to inflation is relatively easy to realize; respectively the annuity indexed to inflation plus a fixed percentage (e.g.: 2 %), if the provider invests only to bonds with yield indexed to inflation.
2. The investment risks are divided between the client and the provider – the indexation depends on the surplus yield achieved in the previous year. The solution in fact is the return refund technique what is applied by the life insurers in the case of traditional (so not UL) life insurances. In this case the service of the insurer increases every year depending on the investment yield achieved in the given year, though the service may not decrease nominally (so a part of the investment risk of the insurer, who guarantees a part of the yield). The guarantee of the insurer is not to achieve at least 0% yield, but to achieve a higher yield what is calculated in the service, to achieve the so called “technical interest rate”. In these cases the insurer



partly invests into bonds, partly into shares, partly into other type of assets e.g.: into real estates. Traditionally the bond requires a minimum regulation and the shares require a maximum regulation, respectively for other investments in order to restrict how much the insurer may loose with the guarantee, since such losses may finally endanger the solvency of the insurer too. With this solution the rate of indexation can not be computed in advance and it is not possible to give a cheap, good guarantee for this.

3. The annuity provider bears the entire investment risk – the annuity is not indexed, it increases in a pre-determined percentage ratio or in absolute terms. This solution is mainly applied in the Anglo-Saxon countries, but the bankruptcy of a some insurance companies of great history (the most important example is one of the oldest and most prestigious English insurers, the Equitable Life) showed what a dangerous strategy it is (though the Equitable Life was able to fulfill its undertaken obligations despite of its bankruptcy). The point is, that the annuity is given this is not indexed (or rarely: its rate changes in a predictable rate) and the insurers may compete with each other, which of them is able to undertake the given series of payment for the lower fee. Thus the competition – with the expenses – is in undertaking the yield reflected in the technical interest rate that is the insurers compete in the magnitude of the technical interest rate. Though the yield beyond the technical interest rate what is undertaken towards the client is of the insurer and they achieve it as they can. In my opinion the solution can not be applied at the mandatory annuities, not so much because of its dangerousness, but because it is not adequate for the application of indexation of unforeseen rate (e.g. depending on the inflation), what is unavoidable at private pension annuities as part of the safety in old age, in my opinion.
4. The client bears the entire investment risk – the magnitude of each annuity depends on how the provisions are evaluated at the maturity of the maturity of the annuity due – indexation in advance is unpredictable. This solution is the implementation of unit linked insurance facilities on annuities. The client bears the entire investment risk, the payment depends on the actual daily



value of the assets. If the invested assets are secure ones (e.g.: government bonds), this is similar to solution No. 1, though the payments unnecessarily fluctuate as a result of the daily changes in the bond evaluations, what is not compensated by the relatively higher yield. If the invested assets are risky, the high yield must be paid for by the great fluctuation of the annuity dues, what is not affordable in the case of small annuities, like the private pension annuities will be, so this solution can not be suggested in the case of private pension annuities.

In summary I deem that only the first two options are possible in the case of mandatory annuities, therefore below I suppose that a choice can only be made from among these two ones.

2.6.2. A practical expectation in respect of indexation – the guarantee paradox

The Hungarian annuity regulation in force ((7) para 4 of Kr 170/1997) provides that the service must be defined so that the annuity paid by the fund shall be indexed at least to the same extent as the pension paid by the social security system. Obviously this rule follows implicitly the indexation technique of return refund. However with this technique it is not reasonable to raise such expectations (what's more, neither at any of the above indexation techniques), because:

- The expectation itself has no relation to what the annuity can provide²⁹. The annuity may fundamentally indexed in proportion to the investment extra yield – if there is no mortality loss.
- In summary it is an incalculable risk for the service provider, because the change in the pension provided by the Social Security System is very much dependent on the politics, so the indexation of the social security pension is practically unpredictable according to the experiences.

As a result of the above said, either one can not find a provider for the mandatory annuity with these conditions, so the state is forced to set up a central provider, or the providers are forced to tie an irrationally high proportion of the

²⁹ This was specifically true as long as the so called Swiss indexation was in force, that could not be covered by capital market facilities, but even it could have been covered, there had in addition be the political risk, that is the state could divert at any time from this indexation, and actually it frequently did.



annuity premium (that is the private pension fund capital of the clients) for the coverage of an unpredictable political risk, so the annuity would be smaller than, what it could be without this indexation rule. Though such a requirement should not be posed to a central provider either, because it can neither guarantee this yield only by market investments – without the involvement of extra state funding. At the elaboration of the regulations it is reasonable to avoid the inclusion of state funding from the start up.

This example already indicates that contrary to the everyday reasoning minimum requirements, guarantees in respect of yield should be cautiously set in the case of annuity regulations. The so called “yield guarantee” paradox³⁰ in general may be worded according to this as the higher yield we wish to guarantee for the client the lower the actual service will be.

This paradox statement may be understood, if we think about the way the higher required yield is achieved by the provider. If the requirement is relatively modest, the provider would probably conduct a more prudent investment policy than it would conduct without this requirement, so it makes investments (primarily into bonds) that are less volatile but yield less. As a result of this rule the yield would be lower on long term than it would be without this it. If the requirement goes beyond a certain value, if it goes over the yield the prudent investment policy of the provider is not sufficient in itself, on top of it a portion of the premium is used to meet the requirement consequently the service will be even smaller.

So expectations similar to the above ones are not reasonable to be raised in respect of indexation and yield, instead investments into bonds indexed to inflation and indexation to inflation should be required or in the case of indexation based on return refund such regulation should be implemented that makes the above goal achievable but does not demand that the above paradox shall be avoided. This is a requirement of 0% technical interest rate (and expected yield) in the case of

The political risk remains even if the announced rule on indexation is the price index by now (as it is more or less the case in 2010), so the theory is adequate in the actual situation as well.

³⁰ This yield guarantee paradox was worded by András Közek (allianz Hungária Biztosító Zrt. Vice CEO on the 22nd of September, 2006 in a expert meeting dealing with annuities as follows: if the legislator forces the insurance companies to assure the higher guaranteed yields the insurers will be able to assure the lower total yield for their clients, because the higher the guarantee the less risky, it is safe, therefore insurers invest the clients' money into facilities providing lower yield.



indexation technique based on return refund, whatever relation the so developed indexation would have with the indexation in Pillar 1. Nevertheless the 0% technical interest rate means, that the annuity increases at the rate of the entire investment excess yield³¹ (naturally only in the case if the mortality result is 0 or positive), what will probably be anyhow over the indexation of Pillar 1 at the majority of the providers even without this requirement.

2.6.3. The indexing and the position of the insured individuals – self selection due to the technical interest rate

When financial experts design the annuity of Pillar 1, they are inclined to think that the option between the annuity starting from a lower level but indexation by a higher index and annuity starting from a higher level but indexation by a lower index is only the issue of the balance of national budget. The first alternate means a lower burden today, because the burden is pushed to a later date, the second alternate is simply the opposite of the first one. Though this is true in respect of the entire stock, but in the case of the individual annuitants this is a completely different issue. For annuitants with different life expectancy different options are favourable. Therefore option among the different alternates is not a kind of choice of financial value, but it is the issue of which segments are privileged. Option No. 1 means that at the beginning the annuity content is higher but is indexed at a lower level, what is favourable for those segments who will expectedly have a shorter life (men, physical workers, people with lower education, lower income). Option No. 2 is a possibility to start with lower annuity but indexed higher, what is favorable for those having a longer life expectancy (women, intellectual workers, people with higher education, higher income). So the total received annuity stock can be differently maximized by one or the other segment.

This logic is valid not only in respect of Pillar 1, but also in respect of mandatory annuity – primarily via the choice made for the technical interest rate. One may say that the higher initial annuity, lower indexation would give back something to those who receive less favourable annuity due to the unisex table, compared to what they

³¹ Respectively the portion of this, what is due for the client, so at the rate of the entire net investment yield.



would receive with a differentiated table. In addition the higher income segment can better live with the devaluation what is the result of the lower indexation.

If the clients may opt for the different technical interest rate, so if they may choose among providers offering different indexation, most probably those having a shorter life expectancy would go to providers offering higher technical interest rate, those with longer life expectancy would opt for a lower technical interest rate. In summary the composition of the stock annuitants would be different from the calculated one, so the provider offering lower technical interest rate would accumulate a risk loss. It is reasonable to avoid it, therefore considerations must be made whether the regulation allows a competition among technical interest rates (more precisely: competition among different rates of indexation).

However the above problem may be solved without competition among technical interest rates, because one type of interest rate favors one the other type of interest rate favors the other segment, so the impact of the choice is not neutral. The solution may be that in the case of private pension fund annuity we calculate with different technical interest rates (so with different indexation rates) at different points of the maturity: 2,9% until the age of 70, 1,5% until the age of 80, and 0% beyond the age of 80. The interest rate is fixed in this case as well, though it still changes, there is no anti-selection due to the interest rate.³² To a certain rate this is compromise between the above two options, it favors less those who are better off than for example the uniform 0% technical interest rate would do.

³² The solution is based upon the István Hetényi's proposal, who proposed on one of the NYIKA meetings in 2007, that the indexation of the pensions in Pillar I should depend on the age of the insured, the higher the age the indexation should be higher. This solution though is rejected by András Simonovits with the argument that Hegényi, being a pensioner at high age is willing to change the rules driven by selfish motivation therefore it should not be taken seriously. But in my view that the following arguments can support this proposal: 1. the higher the technical interest rate the lower the indexation, so the difference between the living standard of the pensions and the active people may ever be far from each other. This does not cause a problem for those of short life, but for those who live long it does. For them this difference is worthy to be diminished or turned back. 2. according to experience pensioners react by decreasing their consumption what may be fairly easily done in the case of certain goods, since many of the goods are already not on the palette of consumption of the old age people as they grow older and older. However there are also goods the consumption of these even grows with aging especially at a high age. These are primarily medical and care services, pharmaceuticals, so above a certain age the strategy of tightening the consumption does not really work especially with respect to the ever smaller families, where ever less relatives can make up the missing pension or provide care informally.



2.6.4. The role and the rate of the technical interest rate

It is worthy to separately look revise the role of technical interest rates since it is different in the case of diverse investments and in the case of the resulting indexation techniques. I elaborate in details the indexation techniques to be considered in case of the mandatory annuity, then I briefly speak about two other indexation techniques as well.

When all the investment risks are undertaken by the insurer and the annuity is not indexed, the technical interest rate is a factor of competitiveness though it is not visible for the clients, what they look at is the price for which the diverse providers offer the same annuity.

When the whole investment risk is of the client, so in the case of UL annuity, the technical interest rate does not have a role to play in principle, since there is no guaranteed yield here, what is one of the most important characteristic of the technical interest rate. Nevertheless a kind of “interest rate” may be defined here too if we apply the strict “technical” sense, or rather a rate can be defined, the rate of how much the number of annuity units diminish year by year. In the case of UL annuities this rate fundamentally plays the same role as the technical interest rate in the case of traditional insurances, that is the investment yield must be at least as much as that so that the paid annuity does not change from year to year and the as much more it is (see “excess yield”) as much the annuity may increase.

2.6.4.1. Technical interest rate in the case of investing into bonds indexed to inflation

The regular interest in case of inflation indexed bonds is: inflation + fixed percentage (e.g.: 2%). In this case the insurer can practically³³ grant two types of indexation for the client:

1. increases the annuity unit with the inflation (maintains the value at real terms) in each year

³³ Theoretically the insurer may still keep the annuity unit unchanged, may not apply indexation and may also grant indexation below the inflation rate but I do not elaborate on these version since I do not deem them realistic options at the moment.



2. increases the annuity unit with the inflation + fixed percentage (it is not necessary as much as the yield of the bond!) so the value is continuously increased in real terms in each year

In the first case the technical interest rate may be at most as much as the fixed percentage rate within the yield of the bond, hence the insurer may grant at most the entire bond yield as yield for the client. However if the technical interest rate reaches this maximum, there is nothing for the insurer to cover its expenses and its profit than the one off portion of costs calculated at the beginning of the maturity as part of the annuity fee, what must cover everything during the entire duration. Therefore it is probably more useful for the insurer not to exhaust the maximum potential of the technical interest rate, and to determine it at a lower rate than this fixed interest rate. Naturally fee (if indexation is fixed) is a factor of competition among the insurers, what is affected by the portion of the cost and the technical interest rate, what pushes the insurer towards determining ever higher technical interest rate.

In the second case the maximum technical interest rate in the bond yield is fixed beyond the inflation and the difference between the fixed interest rates beyond the inflation may be granted for the client. Theoretically it is possible that the insurer offers the client higher fixed increase beyond the inflation than the yield fixed beyond the inflation. In this case the technical interest rate will be negative.

With such an (regulated) investment policy for the regulation of the technical interest rate such provision is sufficient that the insurer may not offer more for the client than it can achieve in the bond yield. Though in the course of the regulation of the technical interest rate it is to be taken into consideration how the very same regulation defines the indexation rule of the mandatory annuities. This may be done in two ways:

1. Each provider may index the annuity according to one scheme (e.g.: precisely to the inflation)
2. There is a competition among the provider in the indexation of the annuity. The possible options are: to the inflation or to the inflation + a fixed percentage.

In the event of the 2nd option the danger of auto selection must be taken into account, since those with shorter life expectancy would opt for the higher indexation,



the ones with longer life expectancy would opt for the initial lower annuity and the corresponding lower, respectively higher indexation. This makes the insurers opting for the higher indexation loss making. If this is recognized nobody would opt for this, so the market would automatically go for version 1, this means that everybody would precisely index the annuity to the inflation.

2.6.4.2. Technical interest rate in the case of return-refund technique

In the case of return-refund technique the regulator can not manage the technical interest rate as liberally as in the case of the previous indexation technique. Two rules are practical to be applied:

1. The technical interest rate be uniform in the entire market, the regulator shall not allow competition on this field
2. The technical interest rate is as low as possible (e.g.: 0%)

Arguments in favour of fixed technical interest rate:

- In the case of life insurances fixing the eventual upper limit of the technical interest rate is a generally applied regulatory practice, because the competition pushes the insurers towards increasing the technical interest rate and encourages them to endanger their long term interest by regarding the short term ones, by this endangering the sustainability of the service. Therefore definition of the low eventual technical interest rate may be regarded as prudential expediency.
- In the case of a strong market competition it has no significance whether the regulation sets the maximum or fixed technical interest rate because the insurers “stick” to the maximum eventual value as a result of the competition. Their starting point is that the vast majority of the clients prefer the certainty of high starting annuity to the eventual higher increase of the value (because it is dependent on excess yield of uncertain magnitude). (In the case of weak market competition – e.g.: monopoly or colluding oligopoly – the situation is naturally different.)
- Despite of the above should there be different kinds of technical interest rate in the market, this would be appropriate for the selection of insured people according to risk criteria, what would aggravate – altogether steepen



– the calculation. In the case of uniform interest rate this anti-selection effect does not exist.

- The uniform interest rate facilitates shifting from one provider to the other in the rentier phase (if the regulation provides for such option) and helps clients to compare the performance of the different insurers. Comparability also means that the providers can not hide the higher costs by higher technical interest rate.

Arguments for lower technical interest rate:

- A priority target is keeping the real value of annuities, what (with this indexation technique) can be achieved the best with the low technical interest rate, since there is a good chance that the yield goes beyond the inflation in each year. Therefore the lower, thus the 0% interest rate is preferred.
- A similarly important target of annuities is not only to sustain its value but possibly the value increases with time. It is best achievable by the low - the 0% -technical interest rate.
- The 0% technical interest rate may be regarded as safety interest rate from the aspect that if during the maturity it would come to light that the recalculation of annuities becomes necessary because the mortality improved; this interest rate helps the best to avoid the decrease of the annuity service and allows for the adjustment against the excess yield. (Naturally the negative yield can not be excluded).
 - However one must note that this technical interest rate is primarily beneficial for the insured people of expectedly healthier, with longer life expectancy and of higher education.

This latter problem is eliminated by the fixed interest rate what changes during the maturity. At the beginning of the maturity the higher (near to the maximum possible 2,9% what is applied in the case of free market insurances, but it can not go beyond that due to the consistency requirement), then from a certain age of the insured (e.g.: from the age of 70) this decreases let us say to the half, and from an other age (e.g.: age of 80) to 0%. By this at the beginning the annuity starts from a higher level, as if the interest rate would be 0% until the end, but at the relatively



earlier age, what the retired person may still have supplementary income from work this is indexed to a lesser rate, later the rate of indexation – parallel with the decreasing of the interest rate – grows gradually.³⁴

It is worthy to briefly look at the dangers of the free option for technical interest rate in the case of return-refund indexation technique:

- If we suppose a non-rigid market, the single applied technical interest rate will in reality be the maximum and there will be no competition
- If the single applied technical interest rate would not be the maximum, the different interest rates would enforce the selection among the insured people and the calculation is made altogether more difficult, and steepens the annuity.
- The higher the technical interest rate the higher the danger that the annuity continuously loses its real value.
- There is decreasing possibility to make the mortality adjustment against the excess return.
- Should there appear actually more technical interest rate in the market (contrary to the above supposition), then the clients would spread according to their related risk among them: the disciplined (and expectedly) long life clients would opt for the lower technical interest rate (because annuity would not lose its value, and the real value would be the higher of that annuity series), while clients with expectedly shorter life would opt for the higher technical interest rate (because during a restricted period of time, while he is alive, would get the most annuity by this).
- Different technical interest rate aggravate the shifting among the providers (if the regulation allows for that, because the selected annuity model so requires) and at the same time also aggravate for the clients the comparison of performances, as allow for the provider to cover the higher costs with higher technical interest rate.

³⁴ As it is proposed by István Hetényi (see before) in respect of the annuity of the Pillar 1.



2.6.4.3. Comments to the actual Hungarian regulation

Finally two short comments in respect of the technical interest rate regulation of the actual private pension-benefit system. According to (5) §3 Kr. 170/1997: the technical interest rate applied in the annuity provider fund may be at most 1,5% higher than the interest rate resulting from the indexation of the social security pension determined for the actual year. I find this definition fatally faulty due to two reasons:

- The technical interest rate in respect of the already started annuities can not be changed annually; therefore it is not justified to adjust it to such volatile standard. In addition also reasonable that the technical interest rates of annuities starting in different years are similar to each other.
- The so set technical interest rate is almost certainly too high, so it is almost certain that such an investment result, let alone higher one be achieved in the majority of the years, so that it ensures the adequate increase of the annuity. Only for comparison: in the case of the life insurances the possible highest technical interest rate may be 2,9% at most.

2.6.5. Indexing, inflation management respectively in the technical literature

In the technical literature of English language the issue of indexation primarily means inflation management, the indexation to inflation respectively, though there are different indexations in the market as well. This is fundamentally due to the fact that in the Anglo-Saxon countries the most frequent annuity is of unchanged amount, without any indexation the so called “level annuity”, whose major deficiency is the lack of stability in value if the annuitant lives long (Blake [1999]). According to Blake the annuitants generally prefer the “level” annuity, because they tend to underestimate their lifetime. In addition its starting level is roughly 30% higher than the one of the indexed annuity (Blake [1999]).

According to James (editor) [1994] private insurers are often blamed for not giving protection against the inflation. According to him one of the possible solutions of the problem is to index the service to inflation and at the same time to keep the provisions in securities indexed to inflation – or in foreign assets, that are immune to the domestic inflation. Chile is mentioned as an example. He adds that the indexed annuity is naturally lower at the beginning as if it was not indexed; therefore it is not



clear whether the workers would be better off with indexation. Poterba [1994] finds the reason for the lack of indexation (in the American market) in the fact that the Treasury only recently started to issue sovereign bonds indexed to inflation, so annuity insurers can only from now on cover the annuities indexed to inflation. According to Blake [1999] the insurer tries to buy indexed bonds at indexed annuities. Though he writes even in 2006 (about the English market) that the indexed annuity is a new development, this is possible, because the government started to issue long term bonds indexed to inflation. (Blake [2006a]).

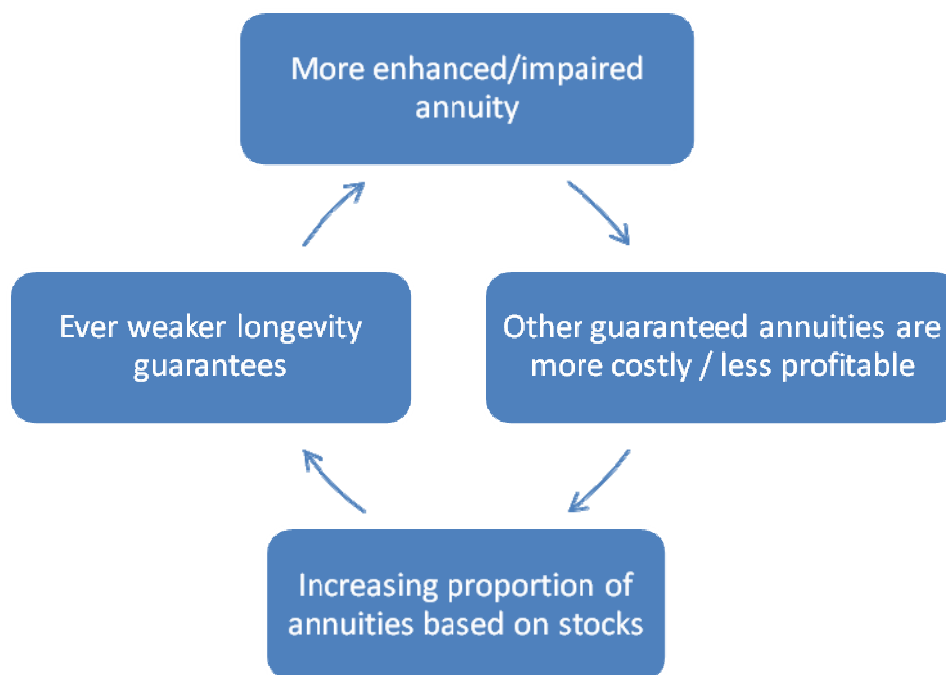
The issue of indexation emerges in relation of UL annuities (as variable, equity linked, etc. annuity) This option has already been mentioned by James (editor) [1994], though not in relation to inflation management, but because in his view this is the solution to the problem of fixing the date of exchange for annuity, since here the price of the annuity units fluctuate together with the market. Nevertheless the disadvantage of this solution is that the risk of fluctuating annuity must be born all along the lifetime of the pensioner. According to Poterba [1997] – though the market of “variable annuity” increased in the market to the highest extent in the past years, though these would not necessarily assure protection against the inflation. He mentions as an example that this may happen in the case if the payments are adjusted to the stock market and that underperforms compared to the inflation.

The UL annuities, as the longevity problems are also considered as a kind of solution from the viewpoint of the insurers. Therefore Wadsworth-Findlater [2002] offers– compared to the traditional “level” annuities - a new kind of annuity of UL type, whose cost structure and internal structure is visible (the English terminology calls it “unbundled”), that permits changes in the course of time and mainly: it does not contain longevity guarantee for the client. The concept was elaborated in by Wadsworth in a brief article in 2002 (Wadsworth [2002]) According to this everybody offers almost identical product in the annuity market (this means here primarily the UK market). This is going to change in the future and he calls this process “unbundling dynamic”. Its logic is as follows:

- The proportion of enhanced and impaired annuities increases – so the pricing becomes more and more sophisticated, what raises the possibility of pricing of the remaining products. (This is called “anti-selection”!).



- This means that the annuity will be more expensive for the survivors, so the traditional annuity covered by bonds makes them less satisfied. So the annuities must shift towards the capital market, towards annuities linked to investments.
- The immediate consequence of this is the less longevity guaranty since this is not adequately priced even now. This dynamics is depicted in the following figure:



Watson Wyatt (whose partner is Wadsworth dealing with annuities) stated in 2008 (Watson Wyatt [2002]) that the UL annuities are more and more popular, but nowadays several insurers suffered a great loss due to these as a result of the non adequate hedging, though the theory of hedging is quite highly developed, and naturally it requires the deep knowledge of the “Greek letters”. Nevertheless the hedging is not enough, but there is a need for the adequate product design and for the periodic monitoring. In respect of product design the combination of adequate incentives (bonuses) and contra-incentives (penalties, exclusions, limitation of the availability of options, etc.) is necessary and the availing of options and customer behavior in respect of deletion must be monitored. In connection with hedging he mentions the basis risk, whose causes in his view are:



- Non-adequate hedging was selected, because there was no better one available
- The fund-manager diverted from the defined track, what is generally allowed for him
- Non-expected costs (e.g. tax)

In the relation with the UL annuities we have to mention that in the Anglo-Saxon insurance market the expression “variable annuity” does not only mean the annuity itself, but also such more or less entirely standard UL insurances, whose theoretical target is that at the end of the maturity the assets accumulated in them are exchanged for annuity. This is what the term means even if the experiences suggest that the majority of people do not exchange the so accumulated money into annuity in the end, since that is not obligatory. This is why a number of publications with the title “variable annuity” practically do not deal with actual life annuities. An example is Abbey-Henshall [2007], who even notes that this term is often interpreted as something neither “variable” nor “annuity”.

The otherwise vague Hungarian annuity technical literature dealt relatively extensively but rather unilaterally with the issue of indexation. Generally the starting point is the provision of the act on private pension fund, which links the indexing of the private pension annuity to the indexing of the social security annuity. Michaletzky [1999] raised but did not really answer the question in connection with this, what the situation is if the yield is not sufficient for indexation. According to the act the fund supervision must be informed, but it is unclear what the supervision is able to do. In contradiction to Michaletzky Réti [Réti 1999] clearly (and correctly) notes that the indexation requirement provided by the law is impractical in insurance technical terms and is contrary to the essence of the of the private pension system. The only indexation that can have an organic relation to the private pension system in his view is the indexation of the no other possibility is available. This also means that pension systems based upon different principles must have different theories for indexation. In this dispute Réti mentions Stahl as a counterpart for discussion and thinks that the understanding of indexation sharply contradicts to regulation 170/1997. Réti’s comments on the social security are also interesting, since he is one of the few people having a profound knowledge of the system in Hungary. According to this the social



security pension system follows the systematic indexation only from the beginning of the 1970-ies, but it was defined in the act only in 1992. Réti gives the formula of the indexation too, from which we learn (without his mentioning it), that 0% technical interest rate is taken as granted. The formula is given in non traditional actuarial formulae, probably because among those he is not confident, since he even says that the classic actuarial formulae can not be used for annuities, because those suppose time without inflation. With such supposition annuity can not be provided in our days.

At this time Stahl does not respond in writing to Réti's proposal, but in the Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] he already notes in respect of Swiss indexation regulation (page 485): "Many expert deems that this provision hides further risks in the system due to the non foreseeable wage increases. It is also unclear, how the insurance companies that provide services can fend off such risks."

However in 2005 he already turns against the regulation in force (Stahl [2005]), namely he says that in his view the basis for the annuity indexation must be the investment and mortality outcome, what may even permit the diminishing of the annuity. This means, that in the indexation he proposes there is no guarantee, consequently there is no need for solvency capital, nevertheless he proposes here, and that in his view a fund even the central annuity provider proposed here may also have solvency capital without any problem.

Arató (Arató [2006a]) disputing with Stahl highlights, that the Hungarian rule on indexation in force in reality is far less certain than what is indicated by Stahl, because the provider does not have to adjust in reality to the Swiss index, what is somewhat more objective, but to the actual raising of the pensions what is very much exposed to the politics and is impossible to anticipate. Therefore he says: (page 274) "I propose that funds and insurers advertise two types of annuities: one without any kind of indexation liability, only recomputed on the basis of the available assets (that means that the annuity may even diminish, this version is basically identical with what János Stahl proposed), the other one based on Swiss indexation. This latter one though would be based upon the actually defined inflation and the wage index. The guarantee fund should operate as a reinsurer too, it would determine fees for the Swiss indexed annuities with regard to other reinsurances and the investment guarantees of the fund. Naturally the starting level of these different annuities would significantly differ;



the annuity without indexation liability may be even 20-30% higher. “Arató supposedly propose two types of annuities, so that there is no pressure on the Swiss indexed annuity, and the starting value would be high, so theoretically the providers are able to manage the Swiss index from the high deduction at the beginning of the term. Naturally the annuity most probably would not be attractive so the other option could provide the way out, what practically could displace the Swiss indexed annuity from the market. So in reality Arató made a proposal on how the Swiss index may formally be maintained – I think in order to make a gesture to politicians who devised and promoted it. We can conclude that practically Stahl and Arató agree.

György Németh joining to the discussion also agrees with such an indexation (Németh [2006], with the significant difference, that contrary to Stahl he thinks about a central pool and not a central provider in respect of mortality, while on the field of investing the provisions the funds would compete with each other. So indexation would be different by funds in each year, but the total of the mortality and investment outcome is common. The problem with this proposal is, that the implementation is probably more problematic, since due to the end-of the year mortality regrouping retrospectively it turns out that the investment yield was not generated for as much provision as it was necessary, so the two outcomes can not be simply added up by funds, due to the mortality regrouping yield regrouping would also be necessary what may easily make this system non transparent. Arató commenting the proposal (Arató [2006] did not pick at this (page 569), but in his view “The author obviously does not know that the mortality of fund members may significantly be diverse in the funds. Following his proposal dangerous trends might start at different funds.” It is not clear what this comment refer to, since if this suggests that fund members fluctuate among funds because of the mortality differences, this is not justified, as Németh started his suggestion with the concept of mortality pool.

We may conclude that in the mid 2000s the technical people greatly rejected the indexation rule of the law, and the common management of investment outcome and mortality result via indexation.

The existing indexation rules are rejected by Barabás-Bodor-Erdős-Fehér-Hamech-Holtzer [2006] and Ágoston-Kovács [2007] joins them. The previously mentioned authors come back to Réti’s proposal according to which “The indexation of



pensions do not have to be necessarily identical, as this is an element of competition for the insured people out of the state system (this also does not comply with the actual rules)".

3. PROBLEM AND MANAGEMENT OF SELECTION PROBLEMS

Premium and reserves formula of annuities (as any other life insurances) assume that the included probabilities are prevalent for the annuities population of the provider. This is true in the following cases:

- The underlying annuities population, or its risk composition is consistent
- As observations concern the past while the calculations concern the future therefore the probabilities may be used with the condition if there is no some kind of trend in the mortality.

This latter phenomenon is called longevity risk the management of which and the mortality tables have already been elaborated. In this chapter though I focus on the first one, how it is achievable that the risk composition of annuitants stock of the different providers is identical with the risk composition of the observed stock, or in other words, how a selection (adverse³⁵ for the provider) in the stock of different providers can be prevented, and should this selection happen what kind of management can be applied.

3.1. POSSIBLE SELECTION EFFECTS AND THEIR MANAGEMENT

3.1.1. Possible selection effects

The problem of adverse (anti- or auto)³⁶ selection may be split into two parts:

1. selection among people buying and not buying annuities
2. selection within people buying annuities

Experience tells that a very strong selection evolved in the voluntary annuity markets, altogether very few people buy annuities though typically these people live longer than the average of the total population. This increases the premium of the annuities, what squeezes out even more people from the annuity market, etc. This phenomenon is a significant obstacle preventing the development of the annuities market. The management of this phenomenon has just recently started by distinction

³⁶ But it is not anti selection. This expression is traditionally not used in the insurance terminology. The distinction between anti and auto selection otherwise suggests an intentional bad faith. If it is the case then we speak about anti-, if not we speak about auto selection. In essence the anti-selection means the evasion of risk assessment. Since in the case of annuitants this does not exist, therefore anti-selection is not relevant here, therefore I call the phenomenon auto selection.



according to health condition (see impaired annuities). Since in this study the precondition is the conversion of mandatory private pension savings into annuities, therefore I focus only on the second type of selection.

The adverse selection within those buying annuity happens mainly due to the following reasons:

- accidentally – this may basically happen at providers / funds with very small stock, that can not present the risk structure of the entire annuitant stock
- due to systemic reasons – this may happen at providers / funds organized on a specific way (e.g.: at different employers, or professions see: “Military Pension Fund”), we may take it as the problem of a closed risk community. Theoretically this may be happen in the case of open risk communities too, but that is either based upon the deception of the clients or that is not lasting as a consequence of the competition.
- Due to the conscious selection of the clients, who prefer those funds that provide parameters are more favorable for them. This selection may happen among funds as well (if the given parameter is applicable for the entire scale of products of the given fund). Naturally selection among the products may also take place in the first two cases what further aggravate the situation. The selection among the products may take place along the following parameters:
 - **By selection amongst the annuity-types.** Here basically important the choice among two parameters of the annuity types:
 - Choice between annuities with guarantee time and without guarantee time, respectively the form of the guarantee time (at the beginning or at the end of the annuity period). The total volume of the annuity in the case of insured people of expectedly different lifetime is different, so the insured people of expectedly shorter lifetime rather choose annuity with guarantee time, the ones of expectedly longer life choose annuity without guarantee time.
 - Choice between annuities for one life or for two lives (provided distinction by sex is prohibited in the premium). In such cases the



woman of a couple is to choose single life annuity, the man of the couple is to choose annuity for two. Theoretically it is possible though in practice it is less probable that the range of products include symmetric and asymmetric two lives annuity. Selection happens if there is an option between these.

- By **selection in the rate of indexation**. The provider can influence it by changing the magnitude of the technical interest rate, so from regulatory point of view this may happen if the technical interest rate (or the magnitude of indexation – see indexation techniques) is not fixed, but it (within certain boundaries) it may be freely selected by the provider. The higher indexation rate (so the higher initial premium, and smaller starting annuity) is rather selected by the people of expectedly longer lifetime, the contrary is selected by those of expectedly shorter lifetime. At cases when selection is made according to the guarantee time the difference compared to the previously discussed cases is that selection is replaced by not the total but the maximizing of gross annuity received during the life.
- **By selecting the starting of the annuity**. If the clients are given the right to start the annuity later than the time of retirement and significant changes are in the possibilities in the hereditary rules before and after the starting of the annuity, it may have a significant selection impact. Then people of expectedly shorter lifetime would tend to delay the starting of the annuity, because if they would die in the meantime the accumulated pension assets would be entirely left for their heir. This significantly deteriorates the mortality profit of the provider. This selection is weakened by the fact, that not every insured person finds legacy important (though it is a fairly general motivation), and also that the starting of the annuity is frequently an economic force for the insured people.



3.1.2. Management of the possible selection effects

For the provider the best method of managing the adverse selection, that is managing the case if the risk composition of the stock of insured diverts in an unfavorable way from the designed one, is the previously discussed distinction. Namely if every risk group pays the premium for the service of the insurer that is adequate for the service, then the composition of the insured stock may divert to any extent from the designed one, the premium paid by them would adequately cover the received services. The problem with the selection – from the insurer's aspect - is not that people of diverse risk get into one risk community, and this is not the primary reason why they should be differentiated by risk, but that the provider does not anticipate the composition of the risk community and can not prepare for the impact of the selection while the regulation prohibits differentiation in general, or according to important aspects (e.g. gender). So problems of selection may be managed by other methods, namely by diminishing the selection, or by the compensation of its effects. The selection has three major areas, which can be managed by different methods:

- 1) **Affecting the entire market.** From among the above methods this means the selection of the starting of the annuity. This may be managed in two ways:
 - a) By prohibiting the delayed starting of the annuity. This may cause the rigidity of the system, what may be eased by allowing the pause right after the starting (though the possibility for inheritance is lost upfront)
 - b) By applying mandatory guarantee time (at the beginning). Then the life annuity portion is a deferred annuity, and during the deferred period the insured person may start the secure annuity when he wants (though he may no further delay the deferred annuity). Practically the OECD model to be discussed later is this solution.
- 2) **Among the providers:** when the risk composition of the annuity stock diverts among the different providers. Among the above selection due to the small stock (if there are several providers in the market at all), selection due to the closed risk community and due to the rate of indexation belongs here. Theoretically it may happen that a provider offers different indexation rates at the same time, but this is not typical. It may also happen that a



provider does not provide every type of annuity from among the ones permitted by the regulation, so the selection within one provider changes into selection among providers. These may be managed by the following methods:

- a) Central provider. This eliminates every selection problem among the providers, as there remains a single provider in the market.

Theoretically this does not manage the problem of the small stock, but in the case of the central provider this may happen only at the starting of the system, and for a short time. The problem is not managed if this central provider offers several types of indexation concurrently.

- b) Making closed all the providers in the period of assets accumulation, and the termination of voluntary movement among them (non voluntary movement, that is movement linked to some other factor is permitted³⁷). Parallel with this every provider must calculate annuity fee for their own mortality conditions. The actual Hungarian solution, which provides that every provider sets annuity calculated for its own mortality conditions, and in the meantime the regulation makes all the funds practically open, is ill-considered, inconsistent, on what good practice can not be built.
- c) Risk equalization (premium clearing), or otherwise unifying the stock, creating a pool among providers. It may be voluntary and mandatory but in certain situation (e.g.: in the case of providers without own capital) the regulator should choose the mandatory pool.

- 3) **Within the same provider:** selection due to the option among different optional annuity products. This may be fundamentally managed with the following methods:

- a) Restriction of the product range, elimination the practical possibilities of option (e.g.: the option is exclusively and only for the single life annuity with guarantee time at the beginning)

³⁷ E.g.: if there is only branch fund, people working in that branch of industry must join, if someone changes work and by this branch of industry the person changes provider as well, but this change is not the reason only the consequence.



- b) Retaining the product range, but the choice of option is terminated, different segments of the risk community is unambiguously assigned to defined annuity products ("segmentation of the risk community"). E.g.: there are single life annuities and for two persons, but the first one is concluded by single people, the second one by couples.
- c) Controlled selection, the different annuity products are calculated by such mortality table, which reflects the risk of those who opted for the given product.

As I prove in the followings this is an illusory option, in practice this can not be implemented.

In summary the following (partly similar, partly different) solutions exist for the selection problems:

- 1) The joint management of the risk of the entire insured stock
 - a) central provider
 - b) by pool
- 2) 2. Restricting the possibilities of choice
 - a) prohibiting the delayed starting
 - b) several types of annuities by clear segmentation of the risk community
 - c) termination of the possibility of changing among providers in the phase of asset accumulation (making the providers closed)
- 3) Restricting the product range, implementation of one type of annuity (e.g. for one insured person guarantee time at the beginning) or
- 4) 4. Controlled selection (as illusory solution)

These, respectively some from among these must be jointly implemented in certain combination.

A more detailed elaboration of the different solutions:

3.2. JOINT RISK MANAGEMENT

3.2.1. Central Provider

There may be different ideas of the central or at least a major provider with functions that on the one hand do not manage selection effects but other problems of



the system³⁸, naturally it would not manage every type of selection problem (e.g. does not manage problems due to products or the starting of annuity). Central providers manage selection risk by not allowing the risk community gets selected among different providers, so both the good and bad risks present themselves at the same provider. General arguments for and against will be later examined in detail when discussing the annuity models in detail, so I only touch upon it from selection aspect.

Theoretically the central provider may be equally established and owned by the state and by private owner (assigned by the state), however the monopolistic position in the case of a single central privately owned (therefore for profit) organization would be difficult to justify, so in reality only the state owned provider may be considered. At the same time one can envisage a central system but not with one provider which manages the selection problems, so those may be for profit, privately owned companies as well. These would have monopolistic situation not in respect of the total stock, only from the aspect of a certain stake of the stock of insured people - e.g.: people buying annuity in a particular year, and would solve the problem of selection. The possibility for its realization is that the government invites a tender for the central provider for the given year. The winner would provide annuity for everyone (naturally based on the mortality projection included in the given cohorts, by this managing the demographic ups and downs – e.g.: Ratkó period) this right would be assigned for a provider in respect of every insured person who retires in the given year until the death of these people, in the consecutive years different other providers would get the same assignment. (Possibly a restriction could be made, that one provider could apply for this tender only in certain periods of time, by this avoiding the development of a real monopolistic situation.) By this the composition of a stock in the hands of a provider would reflect the market average, in addition it would be homogeneous (since everybody retired in the same year.) In addition this is an example that not entire stock of annuities composes a risk community still the majority of selection problems are eliminated. This is the advantage of the solution. The disadvantage is that people retiring in a particular year may get into a worse situation if these people

³⁸ Only as an example: this may be the „final provider” in the very case if nobody would provide annuity service in the market – for example as a result of the rigid rules; such provider to whom the savings of the „disappeared members” could be transferred, etc.



get to a worse provider than the others. An additional disadvantage is that this paves the way for corruption, for example by the providers' collaboration, when they split among themselves who wins in which year.

Experts raised the possibility of a central provider the so called “**model provider**” not yet being in a monopolistic situation. The “model provider” would be such a provider who operates on the one hand as “final provider”; on the other hand it would regulate market by giving alternate against the private insurers. It may play a market regulatory role as well, and would encourage the providers, to provide annuity at possibly maximum level within its range of possibilities. Unfortunately this solution does not manage the selection impacts, what's more it may make the government to conduct not adequately market conform interventions (e.g.: it may keep the annuity premium too low, and the loss is financed from the central budget.), due to this fact this solution would not be really advantageous. Further disadvantage of such a “mixed” solution is – according to a number of experiences – that in such cases either the state takes advantage of its special situation or in case of any trouble the market players delegate the cost of problem management to the state. So such a solution may only be implemented with a kind of suspicious intent that the state displaces the market players in such a situation when there already are market players, but the decision makers want to centralize, though they avoid the direct conflict with the existing market players.

Beyond the above facts in respect of the central provider it is conceivable that centralization is implemented not on the entire annuity system, only on certain elements of that e.g.: annuity payment, management of mortality risk (a central pool would be established), etc. In this case the function of centralization is not necessarily the management of selection effects (with the exception of the central pool), but some other, important aim for example cost saving, convenience of clients, etc.

3.2.2. Stock unification (pool), premium clearing among providers

Stock unification may be voluntary and therefore (probably) partial; this may not concern every provider (though it may happen on voluntary basis too that the entire market unifies the stock); it may also be mandatory and covering the entire market (though theoretically some groups of the market may be forced to unify the stock but



this requires very specific reasons). In the case of voluntary stock unification the risk equalization may reach out to the joint management of investment and mortality risks, but in this situation it is not worthy to maintain separate organizations, and instead of unifying the stock it is more reasonable to unify the organizations. However the mandatory stock unification – fundamentally due to these reasons – may only reach out to the joint risk management, because if the investment risk is jointly managed as well, then we already speak about a central provider. From the other end this means though that the mandatory pool may be considered in such investment and indexation strategy where the risk is in the investment. Namely if the investment into bonds indexed to inflation is mandatory, then (if the need is raised) the central provider is reasonable instead of the mandatory pool, since in this case the “competing” providers may only compete with each other in respect of cost, what is probably a priori lower at the central provider due to the scaled return, than at the different organizations within a system composed of small organizations.

In summary: in respect of mandatory annuities the mandatory stock unification may only cover mortality risk and is worthy to implement with the classical yield-return investment and indexation strategy, otherwise it fundamentally gets into the central provider solution.

That is the stock unification means the joint management of the mortality risk. This may happen in different ways. In its minimal form it means only the profit and loss (risk) equalization generated at the provider when concluding the annuity insurance; in its maximum form it means the continuous distribution, clearing of the profit and loss generated along the entire term.

So the minimal form is a premium clearing mechanism, what would equalize that initial loss and profit that is generated at the different providers as a result of prohibition to apply distinction that is as a result of the mandatory implementation of the unisex premium table. It is almost certain that the gender composition of the new contracts in a certain period of time, e.g.: one year, is different by providers. Naturally it is also conceivable to take other factors e.g.: educational attainment into account. Clearing leads to correct result in the event if the otherwise unisex projected mortality table is correct. Since there is a need for a uniform scale for clearing, therefore in the



case of this mechanism it is necessary that the market has a centrally prepared unisex, projected mortality table. The premium clearing may technically happen so, that the different providers collect the premium (according to the unisex premium table) and they transfer to a central provider the surplus between the actually collected and the premium necessary according to the distinct mortality tables what is initially necessary for provisioning, and the so generated deficit is received from this organization. (This also means that at this mechanism the distinct mortality tables necessary for provisioning must also be unified in the whole market, so these must be centrally defined and implemented mandatorily).

The problem of premium clearing is that the loss and profit generated at the different providers within a period of time would not necessarily offset each other as it is not certain that insured people with favorable and unfavorable characteristics from the insurer's point of view conclude annuity insurance in the same rate at every point of time all over the entire market. If the central mortality projections were correct, then on long term the balance of the profit and loss is neutral, but on short term it is not. This causes problem in practice if a deficit evolved at the central organization in charge of premium clearing (so initial mortality loss generates on the entire market, in other words more people with less favorable conditions from the insurer's point of view conclude annuity contract at a certain point of time). Such transitory deficit may theoretically be covered by the state by repaying it from the transitory surpluses. This may force the state to prepare the mortality projection in a correct manner. Naturally the problem occurs if systematically deficit or surplus is generated at that central organization. The problem with the surplus is that clearly the insured people were deprived from it in a questionable way.

In its minimum form the service providers carry the burden of the mortality losses emerging later or they transfer that (or a part of it) to the clients in the form indexation. It is only logical to distinguish between the initial and the later mortality losses, if the latter one can be transferred to the clients by the indexation (what assumes the return-refund indexation technique. Otherwise one may raise that if a pool is operated it may be done not in a minimum but in a maximum form.



The maximum form practically includes the above minimum form, since the mortality loss of the first year may be largely due to risk composition at the time of concluding the contract. In order to be able to continuously redistribute the mortality losses and gains among the providers, reliable and uniform measurements are needed. Similarly to the premium blending mechanism this requires a centrally designed (standard) provisioning rules and differentiated mortality tables, with a centrally and uniformly defined technical interest rate. Based on these provisioning rules the different providers define the mortality profit and loss in each financial year, and totalize them on market level. If the balance is positive or at least break even then they define the magnitude of the uniform mortality profit per a kind of base, e.g.: per the required provisioning, and where there is a surplus that is transferred to where there is a deficit. If the balance is negative clearly the surplus is not enough to cover that. In this case the deficit is distributed against the return (respectively if the given year was bad in respect of return as well, then against the provision, that is the rate of the annuity) or against the solvency capital function to the fact who stands for the final loss whether the insured person or the provider. Clearly this is in the context of the ownership and capital conditions of the provider. If the owners themselves are the insured and therefore the provider has no own capital, then the loss shall be accounted for the yield, if there is an actual owner so there is solvency capital then the loss may be accounted for that, respectively amounts may be regrouped from where the loss is below the market level (including the profit as well) to where the losses exceed it.

It is worthy to mention that if a central organization is set up on the market for the management of the pool, this attracts the centralization of other element of the annuity service too if this allows cost saving. Finally only the eventual investment remains at the separate providers and they compete in the yield if the rules on indexation permit that. This is exactly the same system as the Swedish private pension system.



3.3. NARROWING THE OPTIONS OF CHOICE

3.3.1. Prohibition of later starting

The possible later starting the payment of the private pension annuity causes adverse selection, if the hereditary rules are different before and after the start of payment of the private pension. In the actual Hungarian regulation such a situation is when before starting the payment of the annuity beneficiary may be assigned for the entire pension capital, though after the starting of the annuity payment heirs do not receive anything – apart from the service due to the guarantee time, respectively the two owners annuity). This problem can be managed by three methods:

1. annuity with mandatory warrant at the beginning or the equivalent regular withdrawal plus deferred annuity facility, where the deferred annuity must be bought at the time of retirement
2. changing the hereditary rule so that only the spouse inherits for his/her pension account, and it is obligatory for married couples to buy two owners annuity so that the other insured person is the spouse. This solution does not manage the problem of single people, but the motive of heritage probably plays a less important role.
3. prohibition of deferred starting of the payment.

This latter solution has such a version that permits, that if the insured person does not yet need the annuity at the beginning of the retired period may further accumulate the money so that later on the annuity would be higher without the problem of adverse selection. This may be achieved by the suspension of the annuity (even immediately after the start of payment).

The suspended annuity behaves as an annuity even during the period of suspension as well, so in the case of death of the insured person there is no more payment (without transferring capital to the heirs). Though the due annuity units are not paid, but those “flow back” to the annuity provision of the insured, by this increasing that and by this continuously increasing the annuity due to the insured. This is exactly why suspension is also in the interest of the insured person, since the longer the period of suspension annuity will be higher at a later age.



3.3.2. Segmenting the risk community

Adverse selection due to choice among the products is also managed without diminishing the product range radically if the risk community is mandatorily segmented according to the products. Adequate justification is necessarily needed for such segmentation. I have identified such a segmentation possibility: if married couples obligatorily buy two owners annuity, and if singles obligatorily buy one owner annuity. With respect that two owners annuity for couples is a “natural unisex situation” for couples, therefore this solution would generally diminish the risk related uncertainty of mandatory unisex annuity, even this makes questionable whether should single people necessarily get unisex annuity? (Though as for my part, I do not go into details concerning this subject.)

Since the basis of this solution is the clear exclusion of segmentation and option for choice (and the consequent adverse selection) therefore the basis for defining the two owners annuity can not be the regular Hungarian practice (what is irrationally and unreasonably libertine), that any kind of relationship is recognized as generating rights identical with marriage, since this practically would bring free choice and by this adverse selection back. This naturally does not mean that the institution of marriage is to what we necessarily have to stick to, that may be a properly documented partner relationship as well, and it only means that segmentation must follow “harder” principles instead of a formal, easily achievable criterion. This solution would also manage the problem, that the mortality of people living in partnership is distinctively different from the one of singles, so this would include an additional new differentiating aspect that stabilizes the calculation.

For the solution the two owners’ annuities must naturally be standardized. One of the possibilities is such a symmetric two owners’ annuity, where one of the insured persons receives 70% of the starting annuity if the other person of the couple dies. The other insured person is strictly and always a married spouse. Depending on the system this may be an annuity with immediately starting payment or deferred annuity. Different two owners’ annuities may naturally be conceived as well, but only one kind should be available in the market.



Generally members of a couple do not retire at the same time. Though when one of them retires then based on the accumulated annuity capital a standard two owners' annuity must be determined for him / her, and when the other member of the couple retires too, the same amount of annuity must be defined for him / her too. The advantage of the solution is that this would solve the problem of survival annuity too, as after the death of one of the spouses the other one would receive $x\%$ ³⁹ of the so far common annuity.

Naturally further detailed rules are needed for the operability of the system that manages the following problems:

- What happens if the retired spouse dies before the other retires?
- What happens if the two owners' annuity payment started and later the spouses divorce?
- What happens if a single person, whose one owner annuity payment already started, would marry?

Possibilities to treat this problem, possible detailed rules respectively (in the order of the above questions):

- The widow simply receives $x\%$ of the annuity of the deceased insured spouse, but the payment may be suspended (it is suspended according to an other rule), until the survivor retires. In such a case the survivor receives the single person's annuity, unless he / she marry again. If the widow dies before retiring the suspended annuity ceases to exist without its payment.
- The annuity is divided into two annuities for single persons (further elaboration is needed to define the proportion, according to one of the possibilities so that they get equal annuity). If one of the spouses did not yet retire the annuity payment is suspended for him / her.
- The annuity is converted into two owners' annuity.

The possible rules of suspension have already been discussed. There is a possibility for the division and unification of annuities in the case if the provisioning

³⁹ We may say that the rational rate of $x\%$ is between 50% or 100%, in between the half and the total value of the common two owners' annuity. The principle is that in the case of a two person household the expenses may be split into three parts: 1. personal expenses of the insured, 2. personal



and the mortality tables used for it as well as the technical interest rates are standardized, so the solution from its part assumes that.

The above regulation does not fit logically to a regulation where the beneficiary can be freely assigned in the phase of accumulation (see the actual Hungarian situation!). Logically a case when the heir of the account is the spouse in the accumulation phase particularly so that the assets of the deceased is credited to the survivor's pension account.

The amendment of the regulation of the accumulation phase would be logical (in order to harmonize it with the above suggested rules in the annuitant phase), so that in the case of divorce the very portion of the balance of the denominated accounts of the couple which was generated in the period the marriage was halved.

3.3.3. Minimizing the possibility of choice between the providers

The issue of development of closed funds (respectively more generally: private pension providers) is raised differently when a private pension system is designed and within the conditions of an existing private pension system. The regulator may a priori decide to establish closed funds e.g.: on technical or on industry branch basis. In this case stable risk communities were created, where the annuity calculation is possible based on own mortality experiences, there was no diverse selection (members are allowed to choose at most among different types of annuities). Though if a fund system has already started in a market, where practically all the funds are open and everybody may enter and leave funds anyway they want, and nevertheless the regulator might still leave providers to develop the mortality table (what the actual Hungarian situation is⁴⁰), when the possibility for fund-choice must be posteriori diminished, so that the system gets nearer to the model of closed funds. Altogether the goal is that the payment of annuity can already start in a stable risk community with computable risk. This does not require the prohibition of every kind of choice in

expenses of the insured, 3. overhead. The overhead roughly remains identical after the death of one of the insured persons.

⁴⁰ Most probably this situation may not have been intentionally generated. It seems so that the funds in Hungary are regulated exclusively based on the characteristics of the accumulation phase, and the rules on the annuity phase are superficial or its characteristics were not taken into account. No selection problem of whatsoever is caused by the free choice of fund of the insured people in the phase of accumulation of assets.



respect of the insured and the providers. The risk community would be stable even if this prohibition is made a certain time before the retirement. Let us say that the insured people may freely move among the funds until the age of 50, but thereafter they may not change and they would receive the annuity from this fund (or from the insurer being in contractual relation with it) until the end of the life (so they may not change in their annuitant period). In this case the provider may calculate annuity for a risk community of stable composition. In such a case it may for example happen that an insurer is related to different funds having diverse risk composition, the insurer provides annuity for each of them, but it does that at diverse premium by funds.

Nevertheless it should be considered whether would it be worthy to make a step backwards from a model based on free option of providers, in order to solve the problem of selection in such a way, or would it be more worthy to choose other methods discussed here for the management of the problem.

3.4. NARROWING THE PRODUCT RANGE

It is obvious that the possibility of choice among different products causes diverse selection. In the followings I examine the mechanism of this diverse selection, and of the maximum loss it causes for the provider. The examination will be made on a choice permitted by the actual Hungarian regulation among immediately starting annuities for singles, so I assume that the insured may choose among three annuities:

1. simple (without guarantee time)
2. guarantee time at the beginning
3. guarantee time at the end

At the beginning of the examination I make a few considerations, and within this I briefly explain the actual, relevant Hungarian regulation too.

3.4.1. Preliminary considerations

3.4.1.1. The Hungarian regulation in force

According to the provisions in force of the Act on Private Pension the annuity service may be:

- classic life annuity
- annuity of front end guarantee time



- annuity of back end guarantee time
- two or more lives annuity

In respect of parameters the situation is:

- no provision of law sets the length of guarantee time , theoretically it may be any long, the client may choose from any value respectively
- in the case of two lives annuity the provision of the law does not provide whether the retirement age, the entitlement for pension respectively must be achieved in order for the payout – theoretically pension may be paid out without entitlement for pension, below the retirement age. The law does not provide either what happens to the annuity following the death of one of the insured individuals. As the act does not separately mention the guarantee time (while it does so in the single life regulation!) therefore we may exclude the two lives guarantee time annuities. I think that those two lives warranties may be excluded where annuity payout stops after the death of one of the insured individuals. However the question is whether the annuity remains unchanged or diminishes after the death of one of the insured. Theoretically both cases are permitted⁴¹.
- In case of annuity for more than two lives – beyond what was said for the two lives, what is applicable here, there is no restriction as to the number of insured individuals.
- Though the Act on Private Pensions does not specify the frequency of the annuity (weekly, monthly, quarterly, etc.) though it may be assumed that it should adjust to the frequency of the annuity from the first Pillar, so it must be monthly. The question still remains whether it should be 12, 13 or eventually more monthly? In the lack of rule we must presume that any solution is possible.

Below I prove that the choice among the products is determined in the above way, its implementation without restriction leads to severe adverse selection what is reasonable to avoid, so the options of choice is reasonable to restrict.



3.4.1.2. The logic of adverse selection as the consequence of selection among products

If looking at the aspects of annuity selection of the clients, two extreme objective functions are imaginable: the client in comparison to the payable premium maximize

1. the payout received during the lifetime
2. all the services.

The reason for this latter one may be the rational consideration that (s)he wants to leave a portion of the capital as a heritage, respectively would like to look after someone (e.g.: the survivor), or the irrational but observable fear that “if I die the insurer swallows my money”. Naturally a transition between these extremes is also possible: if someone maximizes the service received during the lifetime but also wants to look after survivors to a certain extent. However as the guarantee time is not the best form of care for the survivors (as at the time of making the contract one can not know from when to when (s)he must look after someone therefore the solution between these extremes is possibly of very much restricted, and therefore it is reasonable to focus on the extreme objective functions.

The larger the selection problem is if the more people follow this objective function to maximize all the services. There is no selection problem whatsoever if everyone maximizes the service received during the lifetime, since in this case everyone gets simple annuity as nobody finds worthy to pay for the guarantee time to grant annuity for someone after the death. The transitory option between the two clearly mitigates the problems evolving because of this at the provider, what the bigger is if everyone maximizes according to this extreme presumption. Therefore at the computation I use this supposition.

Different types of annuity are worthy to look at in themselves from “potential” adverse selection aspect as well. If we take the simple annuity as the basic case, then we may say that the difference in mortality profit and loss among the best and worse (from annuity point of view) groups of insured (with the shortest expected and longest expected lifetime) is lower at all the other annuity types than in this basic case. Namely:

⁴¹ I think that the growing of the annuity and its diminishing below 50% of its previous value after the death of one of the insured is logically excluded; therefore the range of the possibility is between 50



- The life annuity of front end guarantee time is the total of an annuity certain and a deferred annuity. Within the premium the mortality selection effects only the deferred annuity part, and its weight is smaller – in respect of identical monthly annuity – than in the case of immediate annuities.
- The annuity with back end guarantee time is the total of an immediate life long annuity and a life long life insurance. The annuity and the life long insurance behave contradictory from risk point of view, so the effect of selection is mitigated by the fact that they are present in one modality.

Though this also means that the risk composition of stock of insured is most important at simpler annuities and if we take the annuity with the longer guarantee time its significance is the shorter (though it does not cease to exist) that is the smaller the magnitude of the potential loss due to the unfavorable stock composition. In other words: it would be favorable for the provider if everyone would be obliged to buy an annuity of relatively long guarantee time.

The question from the aspect of adverse selection due to the choice is which group of insured should buy which type of annuity if the objective function of the clients is maximizing all the services compared to the premium paid. The following may be stated:

- The more certain somebody is of not having a long life (e.g.: because he is a man and is ill too), the more worthy for him to buy guarantee time annuity, and the less worthy is to buy single life non-guarantee time annuity, because the guarantee time for him would certainly increase the received service so for him it would be an effective guarantee.
- Those with expected long life (healthy women) would not find worthy to pay the guarantee time surcharge, they are better off to choose single life non-guarantee time annuity as they receive the front end guarantee time service without guarantee time as well, what's more they receive bigger service than the potential value of the entire risk community.

These are only the first ideas in the context of the theme, which will be further elaborated after the following inquiries.

and 100%.



3.4.1.3. Assumptions

Below I rely on the following purely technical presumptions supporting the computation. The findings are not influenced in merits by:

- the annuity is annual and primarily
- g , that is guarantee time is integer. (Not an assumption but a comment, that the value of g mutatis mutandis is either at least 0 (if it is irrelevant whether there is or there is not guarantee time), or at least 1 (if we compare guarantee time annuities, so we have to suppose that there is guarantee time), or 2 (we speak about such front end guarantee time annuities, that guarantees something since the “simple” primary annuity is nothing else than $g=1$ year front end guarantee time annuity)
- g is the same both at front end and back end annuities.

Though the following presumptions affect the substance of the investigation, and are very “strong” assumptions, that are worthy to partially resolve, to mitigate:

- first of all I presume, that the client is precisely aware of his/her remaining lifetime (I mark it by “ h ”). The presumption is necessary to a certain extent, since the essence, respectively the cause of the adverse selection is that the client possesses certain information about his/her expected lifetime, what the provider does not possesses (respectively even if the provider possessed that information could not take it into account). The presumption is an exaggeration, but it also signals what the least favorable situation is from the provider’s aspect. So with this presumption the number received for the impact of the adverse selection may be regarded as the maximum loss.
- Similarly the worst case for the provider is if every single client maximizes all of the services (I mark services by “ sz ”) compared to the premium paid, so they maximize the $\frac{sz}{D_1}$ quotient. A presumption contrary to this is when the client prefers the maximum of the quotient of service during the lifetime and the premium, what is not worthy to investigate, because the result is so much trivial. It is easy to admit that in this case every buys simple annuity as it is not worthy to pay for the guarantee time what is worth nothing to the



client. Every other presumption between these extremes may only be more complex and more biased, what increases the complexity of the analysis, therefore I do not deal with it here, but naturally the issue is worthy for investigation later.

Below I do not have any other presumption for the technical interest rate than it is at least 0%. The formulas become significantly simple if we presume that the interest rate is 0%, therefore I refer to this situation several times⁴². The premium formulas themselves become much simpler as it is shown below. Namely in the case of 0% technical interest rate the different annuity formulas get simplified for the following:

- $\ddot{a}_{x:\overline{h}|} = \ddot{a}_x + g$, as v^k will be 1 for every k
- $A_x = 1$, because the insurance so covers certain events (normally A_x is smaller than 1 only because the technical interest rate above 0%), therefore
- $\ddot{a}_{x:\overline{h}|} = \ddot{a}_x + g$,
- 0,5, so 0,5 higher than the life expectancy at the age of x

It is worthy to take into account the possible magnitude of all the received services (i.e. “sz”). This obviously depends on the type of annuity, the size of guarantee time (g) and the remaining lifetime of the client (h). As above I presumed annual annuities therefore “sz” “leaps” at integer values of h , while the value of h is obviously constant. Therefore the integral part of h $[h]$ plays a great role in the formulas. As the annuity payments take place at different times, therefore for the computation of “sz” the time-value thereof must be taken into account, so the different payments must be reduced to a kind of common denominator. In principle different interest rates may be considered for discounting, but I presume that is only worthy to discount with a technical interest rate. Should we do something else, the result would be very much biased and we may come to almost any conclusion from the calculations. So the possible values of “sz” as a function of the relevant parameters:

- At (“simple”) primary annuity without guarantee time :

⁴² In fact in the first version of this study published in the autumn of 2007 I give a general investigation only of the 0% technical interest rate case – though the calculations were made with different interest rate presumptions. So the below analysis may be regarded as the generalization of the 2007 analysis.



- In general: $sz = 1 + v^1 + v^2 + \dots + v^{h+1} = \frac{1 - v^{h+1}}{1 - v} = \ddot{s}_{\overline{h+1}|i}$,
- with 0% technical interest rate: $sz = [h] + 1$
- annuity of g years with front end guarantee time :
 - in general:
 - if $h < g$:

$$sz = 1 + v^1 + v^2 + \dots + v^{g-1} = \frac{1 - v^g}{1 - v} = \ddot{s}_{\overline{g}|i}$$
 - if $h \geq g$: $sz = 1 + v^1 + v^2 + \dots + v^{h+1} = \frac{1 - v^{h+1}}{1 - v} = \ddot{s}_{\overline{h+1}|i}$,
 - with 0% technical interest rate
 - if $h < g$: $sz = g$,
 - if $h \geq g$: $sz = [h] + 1$
 - with g years back end guarantee time annuity:
 - in general:

$$sz = 1 + v + v^2 + \dots + v^{h+1} + v^{h+1} + v^{h+1} + \dots + v^{h+1+g} = \frac{1 - v^{h+1+g}}{1 - v} = \ddot{s}_{\overline{h+1+g}|i} = \ddot{s}_{\overline{g}|i} + v^g \cdot \ddot{s}_{\overline{h+1}|i}$$
 - with 0% technical interest rate: $sz = [h] + 1 + g$

Below I answer the following questions with the above assumptions:

1. which type of annuity does the insured choose (from among the three ones)?
2. if the insured chooses the guarantee time annuity, which of the possible warranties would he/she choose?
3. what is the rate of adverse selection due to the choice at the insurer?

The answer is given partly by the abstract analysis of annuity-formulas, partly by calculations in such a way that I control the answers given to the third question only by calculations, though the theoretical answers given to the first two questions are controlled by calculations as well. Beyond the above in a separate chapter I investigate



the question whether the adverse selection due to the choice may be eliminated by increasing the premium (or – what is the same – by such a selection table that primarily considers the mortality of those choosing the given annuity).

The investigation is made in two steps in both cases, first for the cases with net premium, and then the results will be generalized for the gross premium cases. Though at the beginning of the investigation I make general considerations concerning the cost portion of the gross premium, respectively of the loading i.e.: the relative value in respect of the net premium, so I make an effort to determine the band of values where it should be.

3.4.2. Which annuity is chosen by the insured – examining the selection effects

3.4.2.1. General considerations – loading thresholds in gross premium cases

In advance and very generally the following can be said about loading

- in the case of different annuities the relative value of the $\frac{SZ}{Dij}$ ratio does not change if the complements are identical, that is if $\lambda^F = \lambda^G = \lambda^H$, the rule of selection among different annuities will be identical in the case of gross and net premium
- as most of the services are provided by the back end guarantee time, the next most by the front end guarantee time and finally by the simple guarantee time (provided that g is identical at both guarantee time annuities and it is not 0, respectively 1), therefore the following inequality must be true for the gross premium:

$$(A_{\overline{g}} + v^g \cdot A_w) \cdot (1 + \lambda^H) > (A_{\overline{g}} + g \cdot A_w) \cdot (1 + \lambda^F) > A_w \cdot (1 + \lambda^F)$$

Of this latter inequality certain thresholds are added on top of the possible magnitude of loadings of different annuities, however the problem of thresholds may be raised more generally, respectively in the case of gross premium - contrary to the problem of net premium – the different types of threshold problem is raised. This problem does not manifest in respect of net premium, the only question to be answered is which of the two or three annuities is worthy to for someone buy, knowing the remaining lifetime and taking granted that the person is interested in



maximizing all the benefits received from the provider. If we presume that everyone knows his/her remaining lifetime, the question of choice is only reasonable if we presume that the clients are obliged to buy annuity from the money they accumulated for that purpose. Should this not be the case the reasonable solution for many of them was not to buy annuity at all, but to budget their money for the remaining lifetime. However in the case of net premium the idea that buying some annuity is not worthy at all (in general, or in comparison to other options) is out of question not only because buying annuity is mandatory but also because the annuity calculation is “equitable”, i.e. it is based on the equation e premium and the expected value of the service.

This is though not the case with gross premium where three further questions may be raised in respect of the absolute and relative magnitude of cost (loading) portion of annuities, in addition to the above discussed problems related to the net premium:

- I. Is it worth at all to buy any annuity i.e. does it not have excessively large loading (apart from whether it is mandatory to buy some annuity)?
- II. Compared to the other annuity types that are alternately optional is it worthy to buy the given type of annuity, i.e. does it not have excessive big loading compared to the other ones?
- III. From what loading value is it at all worthy to buy annuity compared to the other one for a client with a given length of remaining lifetime?

In the first two cases the investigation concerns the “final” limits, i.e. is there anybody at all among the eventual clients of different remaining lifetime (but with identical age at the entry) for whom it is worthy to buy the given type of annuity; in the third case the question is more specific, therefore threshold of III. Type must be within thresholds of type II, and type II within type I. Below I investigate the size of thresholds type I. and II., later I use this for searching thresholds type III. (in some specific cases these may be identical with thresholds type III).

These three questions are raised at all because of including the loading in the cost and obviously its magnitude has reasonable limits. The question is where these limits are.



Let us first examine question I. respectively thresholds type I. The definition of the threshold up to which premium limit it is worthy to buy annuity, i.e. what is the criterion for defining the threshold is not totally obvious. Nevertheless it may be logical to say that if the maximum possible service of an annuity is smaller than its premium it is not worthy to buy. Naturally if it is mandatory to buy an annuity, it is no matter whether it is worthy or not buy a certain annuity, but it is still worthy to be aware of the theoretical upper limit of the price.

The possible maximum service of the annuity is not self evident because its value depends on how long the annuitant of longest life lives. Naturally nobody can tell that, therefore a theoretical value can only be defined. So in order to calculate the maximum possible service first the (statistically relevant) maximum possible age must be decided, by what discount coefficient we shall reduce the annuity units to common denominator. In my view the logical answer for both questions is, that “the coefficient that was taken into account in the calculation”. So the maximum age of the mortality table should be the highest age here too, however we know that there is a (small) chance to have higher actual age than that, and we do not really know what the “biologically possible” maximum age is. For the discounting it is reasonable to take into account that technical interest rate what was used in the calculation, since the client would also be able to make that had he not bought annuity but would manage his/her own assets.

After this the following inequalities may be set up, from these we may determine the threshold of type I.:

- The possible maximum service of the back end guarantee time annuity:

$$1 + v^1 + v^2 + \dots + v^{\omega-x} + v^{\omega-x+1} + \dots + v^{\omega-x+g} = \frac{1 - v^{\omega-x+1+g}}{1 - v} = \ddot{a}_{\omega-x+1+g|}$$

, so i.e. for the gross premium it must come true,

that: $\ddot{a}_{\omega-x+1+g|} > (\ddot{a}_g + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h)$. From this the rule applicable for

$$\text{the complement maximum is: } \frac{\ddot{a}_{\omega-x+1+g|}}{\ddot{a}_g + v^g \cdot \ddot{a}_x} > 1 + \lambda^h \quad \text{MMMM}$$

- The possible maximum service of the front end guarantee time annuity: (provided, that $\omega-x > g$, without whereof in effect we would not mean life



annuity, but annuity certain): $\ddot{a}_{\overline{w-x+1}|}$, i.e. it must come true for the gross premium, that: $\ddot{a}_{\overline{w-x+1}|} > (\ddot{a}_{\overline{1}|} + {}_gP_x \ddot{a}_x) \cdot (1 + \lambda^g)$. From this the rule applicable

for the maximum of the complement: $\frac{\ddot{a}_{\overline{w-x+1}|}}{\ddot{a}_{\overline{1}|} + {}_gP_x \ddot{a}_x} > 1 + \lambda^g$

- The possible maximum service of the simple annuity is the same as the one of the front end guarantee time, so for the gross premium it must be true that: $\ddot{a}_{\overline{w-x+1}|} > \ddot{a}_x \cdot (1 + \lambda^g)$. From this the rule applicable for the maximum

of the complement: $\frac{\ddot{a}_{\overline{w-x+1}|}}{\ddot{a}_x} > 1 + \lambda^g$

In answering question II. the observation made at the beginning of the chapter gives support, According to this there is a strict hierarchy of service among three different single annuity (if we presume, that g is the same at two guarantee time annuities and there is at least 2) strict service hierarchy. According to this the most is provided by the back and, then by the front end guarantee time annuity, and the least is provided by the simple annuity. Therefore the following inequality must exist in respect of gross fees: $(\ddot{a}_{\overline{1}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h) > (\ddot{a}_{\overline{1}|} + {}_gP_x \ddot{a}_x) \cdot (1 + \lambda^g) > \ddot{a}_x \cdot (1 + \lambda^g)$,

In other terms: if these inequalities do not exist, certain type of annuity is not worth the client buying. So the loading of the simple annuity compared to the loading of the front end guarantee time annuity can not exceed a certain level, otherwise only the front end guarantee time annuity is worthy to buy, this situation is the same in respect of simple and back end guarantee time annuities.

Further progress in respect of question II. can be made by exploring the service hierarchy among the annuities:

- From two annuities the one of higher service will be bought by the client if its gross premium is not obviously higher than the gross premium of annuity with service of lower level, i.e. if the loading compared to the loading of the other annuity is too small.
- Nobody will buy though the annuity of higher service if the gross premium is much higher than the provided excess service, i.e. if the loading is too high.



- The above inequalities assist the determination of “too small” loading. Let us look at these, the relative lower limits. If some of the previous three inequalities are not satisfied we may be certain of the result.
- So if $(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^g) \leq \ddot{a}_x \cdot (1 + \lambda^s)$, even if $(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h) \leq \ddot{a}_x \cdot (1 + \lambda^s)$ then nobody buys annuity simple
- if $(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h) \leq (\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^g)$, then instead of front end

guarantee time annuity everybody buys back end guarantee time

Therefore I assume below that:

$(\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h) > (\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^g) > \ddot{a}_x \cdot (1 + \lambda^s)$ comes true. This may

also be noted down in the below alternate forms:, , and $\frac{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x} > \frac{1 + \lambda^g}{1 + \lambda^h}$, or differently

. Naturally it comes true that

, but it is

the consequence of the previous two.

Further considerations are required for the exploration of “too big” loadings. It is obvious that the premium of the “dominant” annuities (so in respect of simple annuity the other two ones, in respect of front end guarantee time annuity the premium of the back end guarantee time annuity) can be increased so much that buying them (in comparison with the annuity as an alternate) becomes a priori irrational for a client. The calculation of the threshold can start from the sz/premium quotient; let us examine the question by annuity pairs. I use the observation in the examination that



annuity of higher service (earlier it was called “dominant”) is not worth for anyone (or at any time) if it is not worth doing neither for those in whose case it provides the much the most services than the annuity providing smaller service. So:

- in respect of simple and back end guarantee time annuities the question i.e. when it will never be worthwhile to buy back end guarantee time annuity, may be interpreted as when it will always come true that:

$$\frac{SZ}{a_x \cdot (1 + \lambda^s)} \geq \frac{SZ}{(a_{\overline{g}|} + v^g \cdot a_x) \cdot (1 + \lambda^h)}$$

Clearly this comes true the most difficultly if the service is a minimum, since the back end guarantee time annuity would be the most worthy for the client. An extreme case of it when the client dies immediately, so he only gets HUF 1 annuity in the simple case and gets F_{g+1} annuity in the back end guarantee time case. In this case there is a maximum difference in the service. So the question is, when the inequality comes true.

$$\frac{1}{a_x \cdot (1 + \lambda^s)} \geq \frac{a_{\overline{g+1}|}}{(a_{\overline{g}|} + v^g \cdot a_x) \cdot (1 + \lambda^h)}$$

Clearly if:

So it can be stated that the inequality

$$\frac{a_x \cdot a_{\overline{g+1}|}}{a_{\overline{g}|} + v^g \cdot a_x} \geq \frac{1 + \lambda^h}{1 + \lambda^s}$$

must come true.

- in respect of the simple and front end guarantee time the insured individual does not buy front end guarantee time annuity as a result of the same consideration, (since the difference in the service of the two annuities is the biggest with the insured individuals of the shortest life expectancy.)

$$\frac{1}{a_x \cdot (1 + \lambda^s)} \geq \frac{a_{\overline{g}|}}{(a_{\overline{g}|} + a_x) \cdot (1 + \lambda^e)}$$



otherwise:

So the inequality

$$\frac{a_{\overline{w}} - a_{\overline{g}}}{a_{\overline{g}} + v^g \cdot a_{\overline{w}}} \geq \frac{1 + \lambda^g}{1 + \lambda^g}$$

must always come true.

- the difference in the service of the front and back end guarantee time annuity gradually increases in the first g year, then it remains constant after the expiry of the front end guarantee time, respectively it is the biggest in real value in year g . So the question is when does the below inequality come true:

$$\frac{a_{\overline{g}}}{(a_{\overline{g}} + v^g \cdot a_{\overline{w}}) \cdot (1 + \lambda^g)} > \frac{a_{\overline{g+g}}}{(a_{\overline{g}} + v^g \cdot a_{\overline{w}}) \cdot (1 + \lambda^h)} = \frac{a_{\overline{g}} + v^g \cdot a_{\overline{g}}}{(a_{\overline{g}} + v^g \cdot a_{\overline{w}}) \cdot (1 + \lambda^h)}$$

Otherwise:

$$\frac{1 + \lambda^h}{1 + \lambda^g} > \frac{(a_{\overline{g}} + v^g \cdot a_{\overline{w}}) \cdot (1 + v^g)}{a_{\overline{g}} + v^g \cdot a_{\overline{w}}}$$

So the inequality

$$\frac{(a_{\overline{g}} + v^g \cdot a_{\overline{w}}) \cdot (1 + v^g)}{a_{\overline{g}} + v^g \cdot a_{\overline{w}}} \geq \frac{1 + \lambda^h}{1 + \lambda^g}$$

must always come true.

In summary the threshold values are:

The „absolute” maximum of the loading, i.e. the threshold values of type I are:

- in the case of back end guarantee time annuity: $\frac{a_{\overline{w-x+1+g}}}{a_{\overline{g}} + v^g \cdot a_{\overline{w}}} > 1 + \lambda^h$



- in the case of front end guarantee time annuity: $\frac{\ddot{a}_{\overline{g-x+1}|} + {}_g\ddot{a}_x}{\ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x} > 1 + \lambda^g$

- in the case annuity simple: $\frac{\ddot{a}_{\overline{g-x+1}|}}{\ddot{a}_x} > 1 + \lambda^g$

$$1 + \lambda^h > 1$$

Naturally it is logical to assume, that the costs are positive, so

$$1 + \lambda^g > 1$$

$$1 + \lambda^s > 1$$

, and , come true, but henceforward it has no significance.

The relative values of the loading must be between the following extremes (threshold values of type II.):

$$\frac{\ddot{a}_x - \ddot{a}_{\overline{g+1}|}}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$$

$$\frac{\ddot{a}_x - \ddot{a}_{\overline{g}|}}{\ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x} \geq \frac{1 + \lambda^g}{1 + \lambda^s} > \frac{\ddot{a}_x}{\ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x}$$

$$\frac{(\ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x) \cdot (1 + v^g)}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^g} > \frac{\ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$$

As it certainly comes true, that $\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x > \ddot{a}_{\overline{g}|} + {}_g\ddot{a}_x > \ddot{a}_x$, (at least if this is the actual guarantee time , i.e. $g \geq 2$) therefore it will come true for every type II. lower threshold value, that it is smaller than 1, i.e.: , , .

The question is what will be the criteria of choice from annuities within the above boundaries; respectively can we give a more specific answer to the values of loading, taking into account the longevity of the client as well. ?



3.4.2.2. Choice between the simple and back end guarantee time annuities

3.4.2.2.1. In the case of net premium

The question is when the inequality comes true, and when its inverse comes true

$$1 + v + v^2 + \dots + v^{[n]}$$

$$\text{, or otherwise } \frac{a_{[n]+1}}{a_x} \geq \frac{a_{[n]+1} + a_x}{a_x + v^g \cdot a_x} = \frac{a_x + v^g \cdot a_{[n]+1}}{a_x + v^g \cdot a_x} ?$$

Rearranging the $\frac{a_{[n]+1}}{a_x} \geq \frac{a_x + v^g \cdot a_{[n]+1}}{a_x + v^g \cdot a_x}$ we receive, that if the inequality $a_{[n]+1} \cdot a_x + a_{[n]+1} \cdot v^g \cdot a_x \geq a_x \cdot a_x + a_{[n]+1} \cdot v^g \cdot a_x$ exists, then the inequality is true.

Deducting the equal components and simplifying it by F_g we receive that if:

$$a_{[n]+1} \geq a_x$$

exists, then the client chooses the simple annuity instead of the back end guarantee time annuity.

As we may approximately call $a_{[n]+1}$, as “discounted remaining life”⁴³, and we

may call a_x as “discounted average remaining life” therefore we may summarize (and somewhat liberally) as the boundary of choice between the back end guarantee time and the “simple” guarantee time is always the discounted general remaining life – independently from the magnitude of the guarantee time. Independently from the duration of the guarantee time the client always chooses “simple” annuity in the case of discounted remaining life beyond that, below that the client always chooses back end guarantee time annuity provided the calculation is of net premium and both variations were prepared by the same mortality table. In this case the “simple”

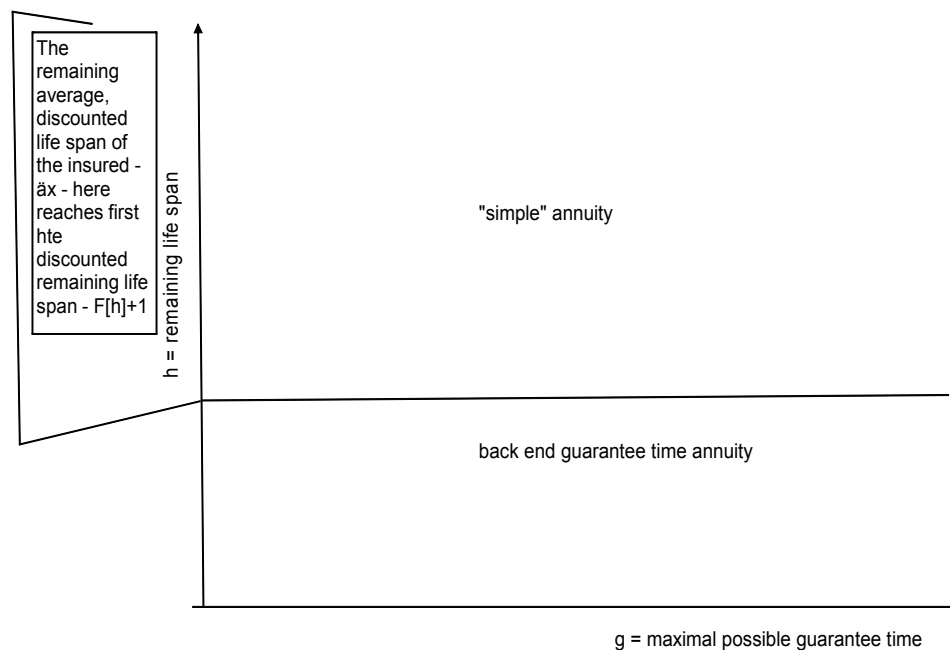
⁴³ As with 0% technical interest rate $a_{[n]+1}$ is simplified to $[n] + 1a$, and a_x to $e_x + 0,5$, therefore the dividing will be $[n] + 1 \geq e_x + 0,5$ inequality. This may be otherwise drawn as



annuity will certainly be loss making, and the back end guarantee time annuity will certainly be profitable for the insurer.

This choice is presented on the below figure:

Figure 1: the choice of the insured individual between the “simple” and the back end guarantee time annuities



3.4.2.2.2. Assuming gross premium

In the gross premium case we do not ask when the inequality is true, and when its inverse is true, but when the

$$\frac{\ddot{a}_{\overline{h}|} + v^g \cdot \ddot{a}_{\overline{h+1}|}}{\ddot{a}_x \cdot (1 + \lambda^g)} \geq \frac{\ddot{a}_{\overline{h+1}|}}{\ddot{a}_g + v^g \cdot \ddot{a}_x}$$

inequality is true, and when its inverse is true.

In order to give the answer let us transform both the gross and the net premium inequality. In the net premium case we may say that the

$[h] \geq e_x - 0,5$. This is a similar, but not precisely identical rule with the remaining life must be longer with the expected remaining life.



$$\frac{a_{\overline{n}|i+1}}{a_x} \geq \frac{a_{\overline{g}|i} + v^g \cdot a_{\overline{n}|i+1}}{a_{\overline{g}|i} + v^g \cdot a_x}$$

inequality, respectively its form transformed into the below

$$a_{\overline{n}|i+1} \geq a_x \cdot \frac{a_{\overline{g}|i} + v^g \cdot a_{\overline{n}|i+1}}{a_{\overline{g}|i} + v^g \cdot a_x}$$

inequality is true if the $a_{\overline{n}|i+1} \geq a_x$ inequality exists.

We may raise the question when the

$$\frac{a_{\overline{n}|i+1}}{a_x \cdot (1 + \lambda^g)} \geq \frac{a_{\overline{g}|i} + v^g \cdot a_{\overline{n}|i+1}}{(a_{\overline{g}|i} + v^g \cdot a_x) \cdot (1 + \lambda^h)}$$

inequality is true, respectively its transformed form the

$$a_{\overline{n}|i+1} \geq a_x \cdot \frac{1 + \lambda^g}{1 + \lambda^h} \cdot \frac{a_{\overline{g}|i} + v^g \cdot a_{\overline{n}|i+1}}{a_{\overline{g}|i} + v^g \cdot a_x}$$

inequality is true?

Since these two inequalities are different from each other only in the $\frac{1 + \lambda^g}{1 + \lambda^h}$ factor, therefore it is true if the

$$a_{\overline{n}|i+1} \geq \frac{1 + \lambda^g}{1 + \lambda^h} \cdot a_x \text{ exists.}$$

So the rule of net premium is modified in the case of gross premium, that the above line is shifted by the ratio of the loading, i.e. above the $a_{\overline{n}|i+1} = \frac{1 + \lambda^g}{1 + \lambda^h} \cdot a_x$ line the client chooses simple annuity, below that he chooses back end guarantee time annuity.

The general considerations tell about the loading, that it has the following “final” cap (thresholds of type I):

$$\frac{a_{\overline{w-x+1}|i+g}}{a_{\overline{g}|i} + v^g \cdot a_x} > 1 + \lambda^h > 1$$

$$\frac{a_{\overline{w-x+1}|i}}{a_x} > 1 + \lambda^g > 1$$

General considerations also show the thresholds of type II as well. These are:

$$\frac{a_x \cdot a_{\overline{g+1}|i}}{a_{\overline{g}|i} + v^g \cdot a_x} \geq \frac{1 + \lambda^h}{1 + \lambda^g} > \frac{a_x}{a_{\overline{g}|i} + v^g \cdot a_x}$$

There is no specific need to determine thresholds of type III because of the simple result; respectively it is trivial, that in respect of upper threshold the inequality



$$\frac{a_x \cdot (a_g + v^g \cdot a_{h+1})}{(a_g + v^g \cdot a_x) \cdot a_{h+1}} > \frac{1 + \lambda^h}{1 + \lambda^g}$$

must exist!

Naturally in respect of (upper) thresholds of type II and type III it exists that type III is at most as much as type II, so

$$\frac{a_x \cdot F_{g+1}}{a_g + v^g \cdot a_x} \geq \frac{a_x \cdot (a_g + v^g \cdot a_{h+1})}{(a_g + v^g \cdot a_x) \cdot a_{h+1}}$$

is true.

This is really true, since by simplification we get the inequality:

$$a_{g+1} \geq \frac{a_g + v^g \cdot a_{h+1}}{a_{h+1}}$$

It is possible with the smallest value, $h=0$ as below:

$$a_{g+1} \geq a_g + v^g = a_{g+1}$$

I.e. it becomes true precisely as equality. By increasing h the left side of the inequality does not change, the right side though decreases, so the inequality exists in the case of every h .

3.4.2.3. The choice between the simple and the front end guarantee time annuities

3.4.2.3.1. In the case of net premium

In this case of front end guarantee time the relation of the service and of the

premium may be $\frac{a_{h+1}}{a_x + a_g}$, and $\frac{a_g}{a_x + a_g}$, depending on how h and g relate to each other.

If $h \geq g$, then $\frac{a_{h+1}}{a_x} \geq \frac{a_{h+1}}{a_x + a_g}$, since $a_x \leq a_x + a_g$, so in this case the client will certainly choose simple annuity – since if he was certain of surviving the guarantee time, why should he pay the price of it, in this case the guarantee time has zero value for him.

If $h < g$, then $\frac{a_{h+1}}{a_x} \geq \frac{a_g}{a_x + F_g}$ will be true in the case if $\frac{a_{h+1}}{a_x} \geq \frac{a_x}{a_x + F_g} \cdot \frac{a_g}{a_g}$ is met, so the client will buy simple guarantee time in the case if the discounted



remaining life of the client is bigger than a part of the (discounted value of the)
guarantee time (the bigger the guarantee time the smaller part).

Since $\frac{a_x}{a_x + a_g} < 1$ is true, therefore $\frac{a_x}{a_x + a_g} < 1$, i.e. $a_x \cdot \frac{a_g}{a_x + a_g} < a_x$, so

the curve $a_{x+1} = \frac{a_x}{a_x + a_g} \cdot a_g$ will always be below the $a_{x+1} = a_x$. However by

increasing g the $\frac{a_x}{a_x + a_g}$ quotient converges to 1, i.e. the curve to the $a_{x+1} = a_x$ line.

However in the case of $h = 0$ we get $g = 1$ from the above equation, so the curve starts from this point on the g axis.

In summary: between the simple annuity and front end guarantee time annuity the client

- chooses the simple annuity if
 - $h \geq g$, i.e. the remaining life is at least as big as the guarantee time (i.e. if the guarantee time does not provide any guarantee for him), and though if
 - $h < g$, but the discounted remaining life is bigger than a part of the

discounted guarantee time, i.e. the $a_{x+1} \geq \frac{a_x}{a_x + a_g} \cdot a_g$ is true.

- and will choose guarantee time annuity if the (discounted) remaining life is

shorter than that, i.e. if $a_{x+1} < \frac{a_x}{a_x + a_g} \cdot a_g$. In this case clearly $h < g$

will also be true, respectively the correlation $a_{x+1} < a_x$ is also met, as well

the curve $a_{x+1} = \frac{a_x}{a_x + a_g} \cdot a_g$ starts from $g = 1$ point.

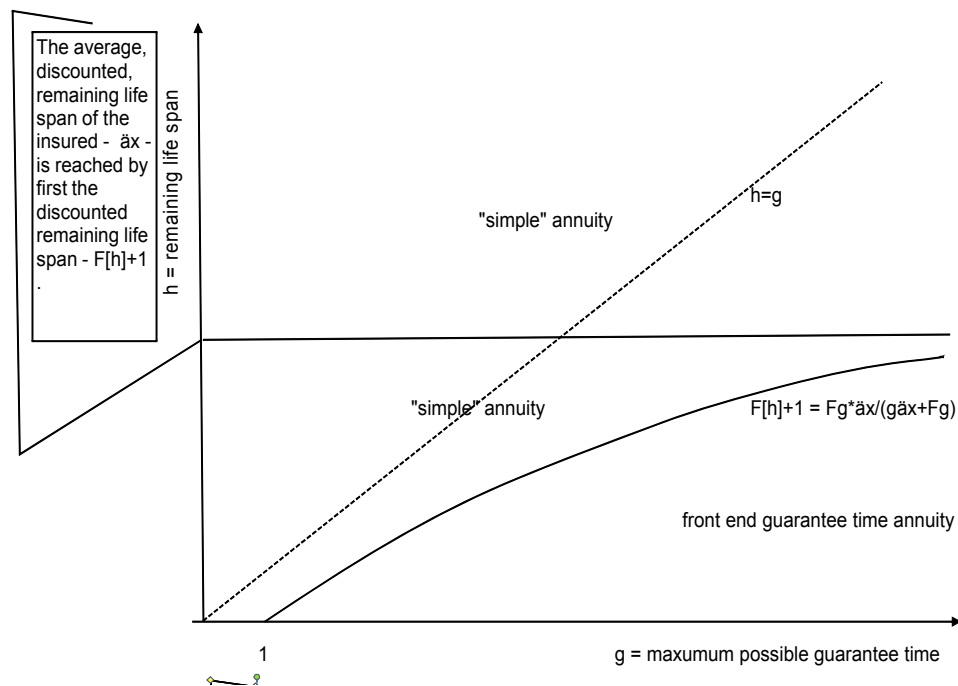
Otherwise this means, that in the $g-h$ coordinate system on the curve (below

the $g = h$ radius) $a_{x+1} = \frac{a_x}{a_x + a_g} \cdot a_g$ and to the points above it the simple annuity belongs, to points below that the front end guarantee time annuity belong, and the



asymptote of this curve is the $\frac{a_{\overline{h}|+1}}{a_x} = a_x$ horizontal line. This can be presented on the below figure:

Figure 2: the choice of the insured person between the simple annuity and the front end guarantee time annuity



3.4.2.3.2. With the assumption of gross premium

General considerations on loading suggest that the following inequality must exist (Type I threshold values):

$$\frac{a_{\overline{w-x+1}|}}{a_g + a_x} > 1 + \lambda^g > 1$$

$$\frac{a_{\overline{w-x+1}|}}{a_x} > 1 + \lambda^g > 1$$

The question may be raised in the case of gross premium that:

1. $h \geq g$ case when does it come true that

$$\frac{a_{\overline{h}|+1}}{a_x \cdot (1 + \lambda^g)} \geq \frac{a_{\overline{h}|+1}}{(a_g + a_x) \cdot (1 + \lambda^g)} \quad ? \text{ (this is when the insured chooses simple annuity)}$$



2. in the case of $h < g$ when it is true that y

$$\frac{a_{\overline{h}|+1}}{a_x \cdot (1 + \lambda^g)} \geq \frac{a_{\overline{g}|}}{(a_{\overline{g}|} + a_x) \cdot (1 + \lambda^g)}? \text{ (the insured individual chooses simple annuity in this case too)}$$

As with the examination of the simple and back end guarantee time annuity, here we may also state that the answer depends on the ratio, and it may also have 3 values:

1. too small, in this case the insured individual will certainly not buy simple annuity and
2. too big, in this case the insured individual will certainly not buy front end guarantee time annuity, but would buy simple annuity,
3. it is between the two thresholds, when further considerations are needed.

The first two questions may be raised in two different ways: generally, i.e. would an annuity with loading of such magnitude be at all good for someone (type II thresholds), and specifically, i.e. the annuity with the given loading is adequate for a client with the given remaining life (type III thresholds)?

The answer given for the first two questions is known from general considerations, i.e. the two thresholds of types II:

$$\frac{a_x \cdot a_{\overline{g}|}}{a_{\overline{g}|} + a_x} > \frac{1 + \lambda^g}{1 + \lambda^s} > \frac{a_x}{a_{\overline{g}|} + a_x}$$

It is certain that any client would buy simple annuity below the bottom threshold, and everybody would certainly buy it above the upper threshold.

Let us see, whether the characteristic that in the case of $h \geq g$ the insured individual would certainly buy simple annuity? In this case we examine the existence of

$$\frac{a_{\overline{h}|+1}}{a_x \cdot (1 + \lambda^g)} \geq \frac{a_{\overline{g}|+1}}{(a_{\overline{g}|} + a_x) \cdot (1 + \lambda^g)}$$

inequality with the condition, what is the same inequality. (The type II upper threshold will certainly not be effective here, since there is no need for the high loading of the front end guarantee time to make the client prefer the simple annuity.) So it can be stated that in the case of gross premium the characteristic of the net premium version remains that in the case of $h \geq g$ the



insured individual will always choose the simple annuity – naturally if \bar{a}_x exists. I.e. in the case of $h \geq g$ the insured individual always chooses simple annuity, if the gross premium of the simple annuity does not exceed the gross premium of the front end guarantee time annuity. Otherwise this also means, that in the case of $h \geq g$ there will not be special effective thresholds of type III, because already in the case of type II bottom thresholds everybody would buy simple annuity.

Consequently the boundary between the choice of the two annuities and the thresholds of type III must be looked for at the case of $h < g$. The type III bottom threshold will be identical with type II bottom threshold. Namely if $h < g$, then as g is an integer, therefore it means that $[h] \leq g - 1$, i.e. $[h] + 1 \leq g$, i.e. $\bar{a}_{[h]+1} \leq \bar{a}_g$.

One of the consequences of this is that
$$\frac{\bar{a}_{[h]+1}}{\bar{a}_x \cdot (1 + \lambda^g)} \geq \frac{\bar{a}_g}{(\bar{a}_g + \bar{a}_x) \cdot (1 + \lambda^g)}$$

inequality, or otherwise the
$$\bar{a}_{[h]+1} \geq \bar{a}_g \cdot \frac{\bar{a}_x \cdot (1 + \lambda^g)}{(\bar{a}_g + \bar{a}_x) \cdot (1 + \lambda^g)}$$
 inequality may only exist

due to this (there is only a possibility for this to exist at all) if
$$1 > \frac{\bar{a}_x \cdot (1 + \lambda^g)}{(\bar{a}_g + \bar{a}_x) \cdot (1 + \lambda^g)},$$
 what if rearranged is precisely the type II bottom threshold: .

When looking for the type III upper threshold we must give an answer to the question, whether the quotient $\frac{\bar{a}_{[h]+1}}{\bar{a}_x \cdot (1 + \lambda^g)}$ has such a high value, above which the client would certainly buy simple annuity, i.e. when the inequality

$$\frac{\bar{a}_{[h]+1}}{\bar{a}_x \cdot (1 + \lambda^g)} \geq \frac{\bar{a}_g}{(\bar{a}_g + \bar{a}_x) \cdot (1 + \lambda^g)}$$

is certainly met? After having rearranged this we receive that

Since it is true that $\bar{a}_{[h]+1} \leq \bar{a}_g$, i.e. that $\frac{\bar{a}_g}{\bar{a}_{[h]+1}} \geq 1$, therefore this type III upper

threshold will certainly be higher than the bottom one, though it also depends on the relation between h and g . It is also certain that in the case of $h \geq g$ we regain the decision making situation of net premium, when though we know that it is not decided by definition that the client buys simple annuity, so we may say for the upper threshold that



$$\frac{A_{\overline{g}|}}{A_{\overline{h}|+1}} \cdot \frac{A_w}{A_{\overline{g}|} + gA_w} \geq \frac{1 + \lambda^g}{1 + \lambda^g}$$

and otherwise:

$$\frac{A_{\overline{g}|}}{A_{\overline{h}|+1}} \cdot \frac{A_w}{A_{\overline{g}|} + gA_w} > 1$$

So we are only looking for the border line.

If we presume that we are within the bottom threshold III, then the borderline between the simple and the front end guarantee time annuity compared to the net premium case is modified so, that not the points above and below the

$A_{\overline{h}|+1} \geq A_w \cdot \frac{A_{\overline{g}|}}{A_{\overline{g}|} + gA_w}$ curve, but its variation extended by the factor, i.e. the points below and above

$$A_{\overline{h}|+1} = A_w \cdot \frac{A_{\overline{g}|} \cdot (1 + \lambda^g)}{(A_{\overline{g}|} + gA_w) \cdot (1 + \lambda^g)}$$

are being looked for.

In the case of net premium this curve was always below the $h = g$ radius, the question is whether this is met now too?

Yes, because if it is met that , then from this the consequence by rearrangement and by complementing it with $A_{\overline{g}|}$ is that

$$A_{\overline{g}|} > A_{\overline{g}|} \cdot \frac{A_w \cdot (1 + \lambda^g)}{(A_{\overline{g}|} + gA_w) \cdot (1 + \lambda^g)}. \text{ If it is met, that , then}$$

$$A_{\overline{h}|+1} = A_w \cdot \frac{A_{\overline{g}|} \cdot (1 + \lambda^g)}{(A_{\overline{g}|} + gA_w) \cdot (1 + \lambda^g)}$$

curve will always be below the $h = g$ radius, and it never intersects or is never tangential to it.

The question is: will this curve also have assymtote, as in the case of net premium?

Let us break the interpretation domain into three parts:

1. if ,
2. if , és
- 3.



In case 1, as it is true that $\ddot{a}_{\overline{g}|} < \ddot{a}_{\overline{g}|} + g\ddot{a}_x$, therefore it is also true that

$$\ddot{a}_x \geq \ddot{a}_x \cdot \frac{1 + \lambda^g}{1 + \lambda^g} > \ddot{a}_x \cdot \frac{\ddot{a}_{\overline{g}|} \cdot (1 + \lambda^g)}{(\ddot{a}_{\overline{g}|} + g\ddot{a}_x) \cdot (1 + \lambda^g)}, \quad \text{i.e. in this case}$$

$$\ddot{a}_{[n+1]} = \ddot{a}_x \cdot \frac{\ddot{a}_{\overline{g}|} \cdot (1 + \lambda^g)}{(\ddot{a}_{\overline{g}|} + g\ddot{a}_x) \cdot (1 + \lambda^g)} \text{ curve not only that the } \ddot{a}_{[n+1]} = \ddot{a}_x \text{ line, but also in}$$

the points above the curve the below the line $\ddot{a}_{[n+1]} = \ddot{a}_x \cdot \frac{1 + \lambda^g}{1 + \lambda^g}$ simple annuity is chosen. If the loading of the simple annuity is smaller than the one of the front end guarantee time annuity, then this curve and its confining line will proportionally be smaller than the curve received in the case of net premium.

In case 2 i.e. when the two loadings are identical, this curve is identical with the curve which was the borderline of choice at the net premium cases, and its confining line will be $\ddot{a}_{[n+1]} = \ddot{a}_x$ exactly as in the case of net premium.

In case 3 (i.e. if the loading of the simple annuity is bigger than the one of the front end guarantee time annuity, but the gross premium of the simple annuity is smaller than the gross premium of the front end guarantee time annuity), it comes

$$\text{true that } \ddot{a}_x \cdot \frac{\ddot{a}_{\overline{g}|} \cdot (1 + \lambda^g)}{(\ddot{a}_{\overline{g}|} + g\ddot{a}_x) \cdot (1 + \lambda^g)} > \ddot{a}_x \cdot \frac{\ddot{a}_{\overline{g}|}}{\ddot{a}_{\overline{g}|} + g\ddot{a}_x}, \text{ so the confining curve will be}$$

$$\text{above the net premium } \ddot{a}_{[n+1]} = \ddot{a}_x \cdot \frac{\ddot{a}_{\overline{g}|}}{\ddot{a}_{\overline{g}|} + g\ddot{a}_x} \text{ confining curve.}$$

$$\text{If } \frac{\ddot{a}_{\overline{g}|} \cdot (1 + \lambda^g)}{(\ddot{a}_{\overline{g}|} + g\ddot{a}_x) \cdot (1 + \lambda^g)} \rightarrow \ddot{a}_{\overline{g}|}, \text{ i.e. the assymtote of the will be the } h = g \text{ radius.}$$

So in summary in the case of gross premium the borderline of choice between the two types of annuity will be similar to the one of the net premium with the condition that the curve is corrected by the ratio of the loading, and in the case of extreme magnitude of loading only the choice of either one or the other annuity is a rational decision. II. (The external borders of the inequality) and type III thresholds (inside of the inequality) will be the following:



$$\frac{a_x \cdot a_g}{a_g + a_x} \geq \frac{a_g}{a_{[h+1]}} \cdot \frac{a_x}{a_g + a_x} > \frac{1 + \lambda^g}{1 + \lambda^g} > \frac{a_x}{a_g + a_x} = \frac{a_x}{a_g + a_x}$$

It is fairly clear that it will meet type II and type III upper cap that

$$\frac{a_x \cdot a_g}{a_g + a_x} \geq \frac{a_g}{a_{[h+1]}} \cdot \frac{a_x}{a_g + a_x}$$

, since by simplifying the inequality we receive that

$$1 \geq \frac{1}{a_{[h+1]}}$$

what is met even at $h = 0$.

3.4.2.4. The choice between the back end and front end guarantee time annuity

3.4.2.4.1. In the case of net premium

In the case of guarantee time annuities we may assume, that g is at least 1. In this case the relation between the service and the premium is:

- in the case of front end guarantee time annuity may be $\frac{a_{[h+1]}}{a_g + a_x}$, and

$$\frac{a_g}{a_g + a_x}, \text{ depending on the relation between } h \text{ and } g.$$

- in the case of back end guarantee time annuity this is always $\frac{a_{[h+1+g]}}{a_g + v^g \cdot a_x}$

Let us see which and when is chosen by the client.

If $h \geq g$, then

$$\frac{a_{[h+1]}}{a_g + a_x} < \frac{a_g + v^g \cdot a_{[h+1]}}{a_g + v^g \cdot a_x} \text{ will be true if it exists:}$$

$$a_{[h+1]} \cdot (a_g + v^g \cdot a_x) < a_g \cdot (a_g + a_x) + v^g \cdot a_{[h+1]} \cdot (a_g + a_x). \quad \text{Having}$$

rearranged this we receive:

$$a_{[h+1]} \cdot (a_g + v^g \cdot a_x) - v^g \cdot a_{[h+1]} \cdot (a_g + a_x) < a_g \cdot (a_g + a_x), \text{ i.e.:}$$

$$a_{[h+1]} \cdot (a_g + v^g \cdot a_x - v^g \cdot a_g - v^g \cdot a_x) < a_g \cdot (a_g + a_x). \text{ Since}$$



$0 < \ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x - v^g \cdot \ddot{a}_{\overline{g}|} - v^g \cdot \ddot{a}_x = \ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}$ is always true, if the guarantee time g is bigger than 0, therefore we may divide by it, i.e. we receive that in this case:

$$\ddot{a}_{\overline{h+1}|} < \frac{\ddot{a}_{\overline{g}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{g}|})}{\ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}}$$

i.e. in the case of $h \geq g$ the client buys back end guarantee time annuity is the above inequality comes true.

It is also interesting how the

$\ddot{a}_{\overline{h+1}|} = \frac{\ddot{a}_{\overline{g}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{g}|})}{\ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}}$ curve relates to the $\ddot{a}_{\overline{h+1}|} = \ddot{a}_x$ line. If $g = 1$, then

$\frac{\ddot{a}_{\overline{g}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{g}|})}{\ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}} = \frac{\ddot{a}_{\overline{1}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{1}|})}{\ddot{a}_{\overline{1}|} \cdot (1 - v) + v \cdot \ddot{a}_{x:1}}$, where $\ddot{a}_{\overline{1}|} = 1$, $\ddot{a}_x + \ddot{a}_{\overline{1}|} = \ddot{a}_x$, and, so

$$\frac{\ddot{a}_{\overline{1}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{1}|})}{\ddot{a}_{\overline{1}|} \cdot (1 - v) + v \cdot \ddot{a}_{x:1}} = \frac{1 \cdot \ddot{a}_x}{1 \cdot (1 - v) + v} = \ddot{a}_x$$

If $g > 1$, then $\ddot{a}_x + \ddot{a}_{\overline{g}|} > \ddot{a}_x$, and $\ddot{a}_{\overline{g}|} > \ddot{a}_{x:g}$, i.e.

$\frac{\ddot{a}_{\overline{g}|} \cdot \ddot{a}_x}{\ddot{a}_{\overline{g}|}} - \ddot{a}_x < \frac{\ddot{a}_{\overline{g}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{g}|})}{\ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}}$, so the $\ddot{a}_{\overline{h+1}|} = \frac{\ddot{a}_{\overline{g}|} \cdot (\ddot{a}_x + \ddot{a}_{\overline{g}|})}{\ddot{a}_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot \ddot{a}_{x:g}}$ curve is

always above the $\ddot{a}_{\overline{h+1}|} = \ddot{a}_x$ line.

If $h < g$, then $\frac{\ddot{a}_{\overline{h+1+g}|}}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x} = \frac{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_{\overline{h+1}|}}{\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x} > \frac{\ddot{a}_{\overline{g}|}}{\ddot{a}_x + \ddot{a}_{\overline{g}|}}$, comes true if it exists

that:

$v^g \cdot \ddot{a}_x \cdot (\ddot{a}_{\overline{h+1+g}|} + 1) > \ddot{a}_x \cdot (\ddot{a}_{\overline{g}|} + 1) \cdot (\ddot{a}_{\overline{h+1}|} + v^g \cdot \ddot{a}_x) / (\ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x) + (\ddot{a}_{\overline{h+1}|} \cdot \ddot{a}_x) - \ddot{a}_x \cdot (\ddot{a}_{\overline{g}|} + 1)$, i.e.

$$\ddot{a}_{\overline{h+1}|} > \ddot{a}_{\overline{g}|} \cdot \frac{v^g \cdot \ddot{a}_x - \ddot{a}_x}{v^g \cdot \ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$$

Since $h < g$ means that $h+1 \leq g-1$, i.e., that $h+1 \leq g$, so that $\ddot{a}_{\overline{h+1}|} \leq \ddot{a}_{\overline{g}|}$, therefore this may only come true if it is true that:

$1 > \frac{v^g \cdot \ddot{a}_x - \ddot{a}_x}{v^g \cdot \ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x}$, so if $v^g \cdot \ddot{a}_{\overline{g}|} + v^g \cdot \ddot{a}_x > v^g \cdot \ddot{a}_x - \ddot{a}_x$. This though is

always true since:



in the case of $g = 0$ the inequality has the form of i.e. this clearly comes true.

In the case of $g = 1$ it has the form of $v + v \cdot {}_1A_x > v \cdot A_x - {}_1A_x$. In this case the left side may take the form $v + v \cdot {}_1A_x = v \cdot (1 + {}_1A_x) = v \cdot A_x$ and from here it can be seen that it is met in the case too.

In the case of $g = 2$ though as $v^2 \cdot A_x + v^2 \cdot {}_2A_x > v^2 \cdot A_x$, i.e. $A_x + {}_2A_x > A_x$ always exists, if g is at least 2, therefore the inequality will always be true.

In summary it is not impossible that the

$A_{[h+1]} > A_x \cdot \frac{v^g \cdot A_x - {}_gA_x}{v^g \cdot F_g + v^g \cdot {}_gA_x}$ inequality is met. The client, in the case of $h < g$ chooses back end guarantee time annuity instead of front end guarantee time one if

$A_{[h+1]} > A_x \cdot \frac{v^g \cdot A_x - {}_gA_x}{v^g \cdot A_x + v^g \cdot {}_gA_x} = A_x \cdot \frac{A_x - \frac{{}_gA_x}{v^g}}{F_g + {}_gA_x}$. Otherwise the client chooses front end guarantee time annuity.

The question is how the $A_{[h+1]} = \frac{A_x - \frac{{}_gA_x}{v^g}}{F_g + {}_gA_x}$ curve relates to the $A_{[h+1]} = A_x$ line. Will it be true that ? Yes, it is true, since it is always true for the numerator of the right

side that , and for the denominator that $A_x + {}_gA_x > 1$, i.e. this curve will always be below the $A_{[h+1]} = A_x$ line.

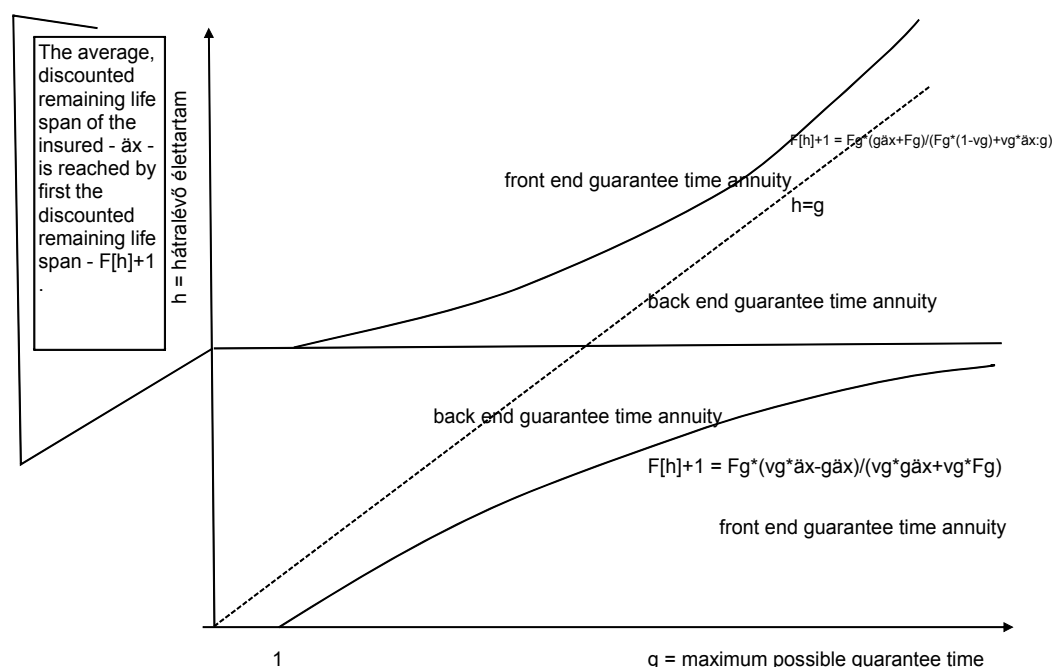
In summary the client chooses back end guarantee time annuity if h is the following band:

$$\frac{A_x - \frac{{}_gA_x}{v^g}}{A_x + {}_gA_x} < A_{[h+1]} < \frac{{}_gA_x \cdot (A_x + {}_gA_x)}{{}_gA_x \cdot (1 - v^g) + v^g \cdot A_{x:g}}$$

and chooses front end guarantee time annuity outside of this band. The upper limit of the band is always above the $A_{[h+1]} = A_x$ line, the bottom limit will be below the line.



Figure 3: the choice of the insured individual among the back end and front end guarantee time annuities



3.4.2.4.2. With the assumption of gross premium

The type I thresholds are the following:

$$\frac{\ddot{a}_{w-x+1|g}}{\ddot{a}_{g|} + v^g \cdot \ddot{a}_x} > 1 + \lambda^h > 1$$

$$\frac{\ddot{a}_{w-x+1|g}}{\ddot{a}_{g|} + g \cdot \ddot{a}_w} > 1 + \lambda^g > 1$$

The type II thresholds are the following:

$$\frac{(\ddot{a}_{g|} + g \cdot \ddot{a}_w) \cdot (1 + v^g)}{\ddot{a}_{g|} + v^g \cdot \ddot{a}_w} > \frac{1 + \lambda^h}{1 + \lambda^g} > \frac{\ddot{a}_{g|} + g \cdot \ddot{a}_w}{\ddot{a}_{g|} + v^g \cdot \ddot{a}_w}$$

If $h \geq g$, then in the case of net premium the client chooses back end guarantee time annuity, if it is met that

$$\frac{\ddot{a}_{g|} + v^g \cdot \ddot{a}_{w-x+1|g}}{\ddot{a}_{g|} + v^g \cdot \ddot{a}_w} \geq \frac{\ddot{a}_{w-x+1|g}}{\ddot{a}_{g|} + g \cdot \ddot{a}_w}$$

Arranging it to F_{w+1} side we receive that it exists if it is met that:



$$a_{\overline{g}|} + v^g \cdot a_{\overline{h+1}|} \geq a_{\overline{h+1}|} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x}$$

i.e.

$$a_{\overline{g}|} \geq a_{\overline{h+1}|} \cdot \left(\frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x} - v^g \right)$$

Question: is the coefficient of $a_{\overline{h+1}|}$ always positive, can we divide with it? The answer is yes, since:

$$\frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x} - v^g = \frac{a_{\overline{g}|} + v^g \cdot a_x - v^g \cdot a_{\overline{g}|} - v^g \cdot {}_g p_x a_x}{a_{\overline{g}|} + {}_g p_x a_x} = \frac{v^g \cdot a_{x:\overline{g}} + (1 - v^g) \cdot a_{\overline{g}|}}{a_{\overline{g}|} + {}_g p_x a_x}$$

where every coefficient is positive, so the fraction is positive as well. So the solution, that:

$$\frac{a_{\overline{g}|}}{a_{\overline{g}|} + {}_g p_x a_x - v^g} = a_{\overline{g}|} \cdot \frac{a_{\overline{g}|} + {}_g p_x a_x}{v^g \cdot a_{x:\overline{g}} + (1 - v^g) \cdot a_{\overline{g}|}} \geq a_{\overline{h+1}|}$$

In the case of gross premium the question may be raised as when is the

$$\frac{a_{\overline{g}|} + v^g \cdot a_{\overline{h+1}|}}{(a_{\overline{g}|} + v^g \cdot a_x) \cdot (1 + \lambda^h)} \geq \frac{a_{\overline{h+1}|}}{(a_{\overline{g}|} + {}_g p_x a_x) \cdot (1 + \lambda^g)}$$

inequality met?

Also arranging for the $a_{\overline{h+1}|}$ side, we receive that

$$a_{\overline{g}|} \geq a_{\overline{h+1}|} \cdot \frac{1 + \lambda^h}{1 + \lambda^g} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x} - v^g \cdot a_{\overline{h+1}|} - a_{\overline{h+1}|} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^g} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x} - v^g \right)$$

We can arrange to the $a_{\overline{h+1}|}$ side, if it is met that:

$$\frac{1 + \lambda^h}{1 + \lambda^g} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_x}{a_{\overline{g}|} + {}_g p_x a_x} - v^g > 0$$

i.e.

$$\frac{1 + \lambda^h}{1 + \lambda^g} > \frac{v^g \cdot (a_{\overline{g}|} + {}_g p_x a_x)}{a_{\overline{g}|} + v^g \cdot a_x}$$

As according to type II threshold it always exists that

, therefore the above inequality is clearly met, what at the same time means the threshold of type III.



So in the case of gross premium if $h \geq g$, then the client may choose back end guarantee time annuity if it is met that

$$\frac{\frac{A_{\overline{g}|}}{1 + \lambda^h} \cdot \frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} \geq A_{\overline{g}|+1}$$

So the borderline is the

$$A_{\overline{g}|+1} = \frac{\frac{A_{\overline{g}|}}{1 + \lambda^h} \cdot \frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} - v^g$$

curve. As $h \geq g$, therefore this curve is an effective borderline only if it is above the $h=g$ radius. Therefore the question is whether it will always be true if it exists that:

$$\frac{1 + \lambda^h}{1 + \lambda^g} \geq \frac{v^g \cdot (A_{\overline{g}|} + v^g \cdot A_{\overline{g}|})}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}$$

Or otherwise, will it be always true with the condition that

$$A_{\overline{g}|} \leq \frac{\frac{A_{\overline{g}|}}{1 + \lambda^h} \cdot \frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} - v^g$$

Simplifying and rearranging the question is modified to be the following question:

$$\frac{1 + \lambda^h}{1 + \lambda^g} \cdot \frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} - v^g \leq 1$$

respectively:

$$\frac{1 + \lambda^h}{1 + \lambda^g} \leq \frac{(1 + v^g) \cdot (A_{\overline{g}|} + v^g \cdot A_{\overline{g}|})}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}$$

This is though nothing else, then the type II threshold value, i.e. the curve is really always the effective cap, because it is above the $h = g$ radius.

If $h < g$, then in the net premium case the client chooses back end guarantee time annuity if it is met that

$$\frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|+1}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} \geq \frac{A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}$$

Arranging it to $A_{\overline{g}|+1}$ side we get that this exists if it is met:

$$A_{\overline{g}|+1} \geq \frac{A_{\overline{g}|}}{v^g} \cdot \left(\frac{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}}{A_{\overline{g}|} + v^g \cdot A_{\overline{g}|}} - 1 \right)$$



$$\frac{\ddot{a}_g}{v^g} \cdot \left(\frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x} - 1 \right) \text{ may be stated in other form too:}$$

$$\ddot{a}_g(1+g+1)^g / v^g \cdot ((\ddot{a}_g(1+g+1)^g + v^g \cdot \ddot{a}_x) / (\ddot{a}_g(1+g+1)^g + (\ddot{a}_g(1+g+1)^g) \cdot x) - 1) = \ddot{a}_g(1+g+1)^g$$

In the case of gross premium the question is when will the below inequality be true?

$$\frac{\ddot{a}_g + v^g \cdot \ddot{a}_{g+1}}{(\ddot{a}_g + v^g \cdot \ddot{a}_x) \cdot (1 + \lambda^h)} \geq \frac{\ddot{a}_g}{(\ddot{a}_g + \ddot{a}_x) \cdot (1 + \lambda^e)}$$

If we transform it we receive if:

$$\ddot{a}_g + v^g \cdot \ddot{a}_{g+1} \geq \ddot{a}_g \cdot \frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x}$$

$$\ddot{a}_{g+1} \geq \frac{\ddot{a}_g}{v^g} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x} - 1 \right)$$

$$\ddot{a}_{g+1} = \frac{\ddot{a}_g}{v^g} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x} - 1 \right)$$

So above the curve the client always buys back end guarantee time annuity. The question is whether this limit is always effective, i.e. the curve is always below the $h=g$ radius, i.e. is it always true that

$$\ddot{a}_g \geq \frac{\ddot{a}_g}{v^g} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x} - 1 \right)$$

Otherwise this means that the following must exist:

$$v^g \geq \frac{1 + \lambda^h}{1 + \lambda^e} \cdot \frac{\ddot{a}_g + v^g \cdot \ddot{a}_x}{\ddot{a}_g + \ddot{a}_x} - 1$$

i.e.:

Though this is nothing else than the upper cap of type II existing in the case of loading, so the curve is always the effective limit, i.e. it is always below the $h=g$ radius.

Naturally within the existing thresholds of loading, i.e. it is true that

$$\frac{(\ddot{a}_g + \ddot{a}_x) \cdot (1 + v^g)}{\ddot{a}_g + v^g \cdot \ddot{a}_x} \geq \frac{1 + \lambda^h}{1 + \lambda^e} > \frac{\ddot{a}_g + \ddot{a}_x}{\ddot{a}_g + v^g \cdot \ddot{a}_x}$$



In summary the client chooses back end guarantee time annuity is h is in the following band:

$$\frac{\frac{a_{\overline{g}|}}{1 + \lambda^h} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_{\overline{w}|}}{1 + \lambda^g} - v^g}{\frac{a_{\overline{g}|}}{1 + \lambda^g} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_{\overline{w}|}}{1 + \lambda^g} - v^g} \geq a_{\overline{h+1}|} \geq \frac{a_{\overline{g}|}}{v^g} \cdot \left(\frac{1 + \lambda^h}{1 + \lambda^g} \cdot \frac{a_{\overline{g}|} + v^g \cdot a_{\overline{w}|}}{a_{\overline{g}|} + a_{\overline{w}|}} - 1 \right)$$

Outside the above band respectively is the loadings are outside the range defined by the thresholds, the insured individual chooses front end guarantee time annuity.

3.4.2.5. Summarizing: the choice among annuities (in the case of net premium)

If we summarize the above the following statement can be made:

3.4.2.5.1. Summary rules

The result of the above three double examinations is the following three rules.

1. the client chooses between the simple and the back end guarantee time annuity so that above the $F_{h+1} = a_{\overline{w}|}$ line he chooses simple, below it he chooses back end guarantee time annuity
2. the client chooses between the simple and front end guarantee time

annuities so that above the $a_{\overline{h+1}|} = a_{\overline{g}|} \cdot \frac{a_{\overline{w}|}}{F_g + a_{\overline{w}|}}$ curve he chooses simple, below that he chooses front end guarantee time annuity. The

$a_{\overline{h+1}|} = a_{\overline{g}|} \cdot \frac{a_{\overline{w}|}}{F_g + a_{\overline{w}|}}$ curve is always below the $a_{\overline{h+1}|} = a_{\overline{w}|}$ line, as well as

the $a_{\overline{h+1}|} = a_{\overline{g}|} \cdot \frac{a_{\overline{w}|}}{F_g + a_{\overline{w}|}}$ curve starts from the $g=1$ point.

3. the client chooses between the back end and front end guarantee time

annuity so, that between the $a_{\overline{h+1}|} = a_{\overline{g}|} \cdot \frac{a_{\overline{w}|} - \frac{a_{\overline{w}|}}{v^g}}{a_{\overline{g}|} + a_{\overline{w}|}}$ and the

$a_{\overline{h+1}|} = a_{\overline{g}|} \cdot \frac{a_{\overline{g}|} \cdot (F_g + a_{\overline{w}|})}{a_{\overline{g}|} \cdot (1 - v^g) + v^g \cdot a_{\overline{w}|}}$ curves he chooses back end guarantee

time annuity, outside those he chooses front end guarantee time annuities.

The upper limit of the band is always above the $a_{\overline{h+1}|} = a_{\overline{w}|}$ line, the bottom limit will be below the line.



If we apply these three rules together, then we come to the conclusion that:

- according to the rule 1. and 2. the insured individual will anyhow buy simple annuity above the $\ddot{a}_{\overline{n}|i} = \ddot{a}_x$ line. The rule No. 1. and 2. also means, that above the line the simple annuity dominates both the front and the back end guarantee time annuity, so rule No. 3 has no significance.

- Though below the $\ddot{a}_{\overline{n}|i} = \ddot{a}_x$ line, according to rule 1. the back end guarantee time annuity dominates the simple annuity, so below the line the insured individual will certainly choose guarantee time annuity according to the rules No. 2. and 3. According to rule No. 3. the

$$\ddot{a}_{\overline{n}|i} = \ddot{a}_x \cdot \frac{\ddot{a}_x \cdot (\ddot{a}_x + {}_s\ddot{a}_x)}{\ddot{a}_x \cdot (1 - v^s) + v^s \cdot \ddot{a}_{x:s}}$$
 curve is always above the $\ddot{a}_{\overline{n}|i} = \ddot{a}_x$ curve, so this curve does not have an effective role in choice below the line.

- according to rule No. 3. in the case of the area below the $\ddot{a}_{\overline{n}|i} = \ddot{a}_x$

line, above the
$$\ddot{a}_{\overline{n}|i} = \ddot{a}_x \cdot \frac{\ddot{a}_x - \frac{s}{v^s} \ddot{a}_x}{\ddot{a}_x + {}_s\ddot{a}_x}$$
 curve back end guarantee time, below that front end guarantee time will be chosen by the client.

According to rule No. 2. the
$$\ddot{a}_{\overline{n}|i} = \ddot{a}_x \cdot \frac{\ddot{a}_x}{F_s + {}_s\ddot{a}_x}$$
 curve is the limit.

Above it simple annuity, respectively according rule No. 1 the back end guarantee time annuity non dominating this, below that front end guarantee time annuity is chosen by the insured individual. As

$$\ddot{a}_x \cdot \frac{\ddot{a}_x}{F_s + {}_s\ddot{a}_x} > \ddot{a}_x \cdot \frac{\ddot{a}_x - \frac{s}{v^s} \ddot{a}_x}{\ddot{a}_x + {}_s\ddot{a}_x},$$
 therefore the borderline between the front end and back end guarantee time annuities will be the

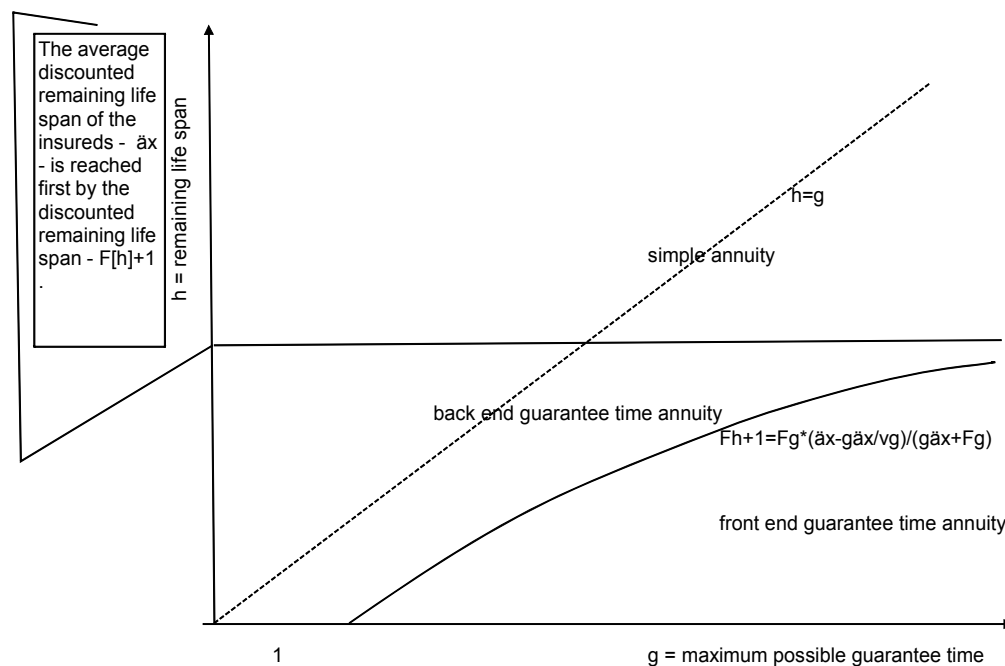
$$\ddot{a}_{\overline{n}|i} = \ddot{a}_x \cdot \frac{\ddot{a}_x - \frac{s}{v^s} \ddot{a}_x}{F_s + {}_s\ddot{a}_x}$$
 curve.



3.4.2.5.2. Summary table

In case No. 1. from among the above 4 curve two will be effective, and those two will divide the g - h plane according to the below manner (I only indicated the $h=g$ radius for the sake or orientation):

Figure 4: the choice of the insured among 3 annuities



3.4.2.5.3. With simple argumentation (in the case of 0% technical interest rate and net premium)

Ignoring the formulas, the above results may be explained by the followings. I assumed 0% technical interest rate at the explanation, because leaving the interest apart transparency is greatly improved:

- If the client knows, that he gets back at least the price of the simple annuity as a service (i.e. his remaining life exceeds the expected value), then for him it is not worth paying for the guarantee time, since the $\frac{F}{Df}$ quotient is already bigger than one. In the case of front end guarantee time annuity, the guarantee time shorter than the lifetime does not offer the client any extra



service for the extra premium, in the case of longer guarantee time than the life expectancy the premium (what is anyhow bigger than the guaranteed service) is certainly bigger than the service. It is neither worthy for him to buy back end guarantee time (HUF 1 extra service), because he can buy every extra 1 year extra back end guarantee time for exactly HUF 1 premium, so the ratio of the extra service and the premium is 1, so it is less favorable, than what he has already achieved with the simple annuity, so it is not worthy to buy such service.

- In general we may say, that if the client lives longer than the possible length of the guarantee time, it is not worthy for him to buy front end guarantee time, as he does not get any extra service for the extra premium paid for the guarantee time.
- If the client knows, that in the case of the simple annuity he does not get back the premium as a service (because the remaining life is shorter than the average), then compared to this it is more worthy for him to buy back end guarantee time annuity than simple annuity. In this case the premium for 1 year extra service is HUF 1, so the ratio of the extra service and the extra premium is 1 what is less favorable higher than the service-premium ratio; so buying one year back end guarantee time increases this ratio. This is true for every extra year, so in this case it is worthwhile to buy the possible highest guarantee time, back end guarantee time annuity. Within this if the possible biggest guarantee time is smaller than the remaining life, it is neither worthwhile to buy the front end guarantee time annuity either, so in this case the optimum choice is the back end guarantee time (of maximum length) annuity.
- Based on the above front end guarantee time annuity – it is rewarding to buy guarantee time annuity at most is the guarantee time is higher than a remaining life, the remaining life though itself is smaller than the average. Though (within this) buying the possible longest guarantee time is rewarding, because the magnitude of the service (the numerator) is identical with the guarantee time, though the denominator is the guarantee time and



“something else”, i.e. the denominator will be smaller than one. This “something else” is nothing else, than the premium of the annuity delayed with the guarantee time. In such a case if we increase the guarantee time by one, then the numerator is increased by one the denominator is increased by less than one, as the increasing of the front end guarantee time by one year increases the premium by less than one (the only thing that happened was that an uncertainly due, so a service of smaller premium than one becomes certain, so it becomes of unit premium service. The ratio of extra premium and extra service is bigger than one, i.e. adding the extra guarantee time increases the quotient of service-premium.

- Though it is not certain that it is not rewarding for the client to buy front end guarantee time annuity. It is certain that the back end guarantee time annuity in this case is better than the simple annuity, but in the majority of the cases it is even better than the front end guarantee time annuity. Supposing that the maximum possible guarantee time is identical in the front end and in the back end guarantee time version, we may say, that if the client lives almost until the length of the guarantee time, then it is more advantageous for him to buy back end guarantee time than front end guarantee time annuity. This can be seen from the fact that the front end guarantee time relatively slightly increases the premium of the annuity compared to the simple annuity, so we may take it as identical. If the service-premium quotient is smaller than one at the front end guarantee time annuities, and in the case of back end guarantee time of the same magnitude both the numerator and the denominator increases roughly by the guarantee time, so the extra service-extra premium quotient is 1, what increases the service-premium quotient compared to the front end guarantee time variation. Though if we decrease the client’s remaining life at a given guarantee time , then after a while we may say that it is not rewarding for the client to pay by the extra premium of the back end guarantee time for the extra guarantee provided by the insurer compared to the front end guarantee time annuity, as while the ratio of the premium of



two types guarantee time annuities is unchanged, the ratio of the guarantee provided by back end guarantee time annuity is ever decreasing, so after a while it will not be rewarding for the client. Should we examine the case, when the guarantee time of the front end guarantee time and back end guarantee time annuity is different then we might say that if the front end guarantee time is longer than the life + back end guarantee time, then it is certainly worthwhile to choose the more expensive back end guarantee time. Even above this it is more rewarding to choose the front end guarantee time annuity for a while, than the back end guarantee time one, but in the end the back end guarantee time annuity “wins”.

3.4.3. How long guarantee time does the insured individual choose?

3.4.3.1. In the case of back end guarantee time annuity

In the case of net premium according to the above the back end guarantee time annuity is only selected by the insured person if the inequality $\frac{a_{\overline{H+1}|} + v^{H+1} \cdot a_{\overline{g}|}}{a_x + A_x \cdot a_{\overline{g}|}} < a_x$ is met. In this case the client must maximize the quotient. The question is whether this is a function which increases or decreases according to g .

If we increase g by one, then the numerator increases by $v^{H+1} \cdot v^g$, the denominator increases by $A_x \cdot v^g$. The question is whether the following inequality will be true:

$$\frac{a_{\overline{H+1}|} + v^{H+1} \cdot a_{\overline{g}|}}{a_x + A_x \cdot a_{\overline{g}|}} < \frac{v^{H+1} \cdot v^g}{A_x \cdot v^g} = \frac{v^{H+1}}{A_x}$$

If we transform it we receive that it is true if it exists that:

$$a_{\overline{H+1}|} \cdot A_x + v^{H+1} \cdot a_{\overline{g}|} \cdot A_x < a_x \cdot v^{H+1} + A_x \cdot a_{\overline{g}|} \cdot v^{H+1}$$

Simplifying it if:

$$a_{\overline{H+1}|} \cdot A_x < a_x \cdot v^{H+1}$$

Since

$$a_{\overline{H+1}|} < a_x$$

therefore this certainly exists, if it is true that



$$A_x < v^{n+1}$$

i.e. if it exists:

$$A_x = 1 - (1 - v) \cdot A_x < v^{n+1}$$

Otherwise

$$\frac{1 - v^{n+1}}{1 - v} = A_{\overline{n}|i} < A_x$$

what is naturally true?

So it is true, that by the increasing of g the service-premium quotient increases, so the client always chooses the biggest one in the case of back end guarantee time from among the possible warranties.

As concern the future we may generalize our observation, i.e. we may state that if it is true that:

$$A_{\overline{n}|i} < A_x$$

then it exists that

$$A_x < v^{n+1}$$

and vice versa, it is true that

$$A_x < v^{n+1}$$

then it will be true that

$$A_{\overline{n}|i} < A_x$$

as

$$A_x = 1 - (1 - v) \cdot A_x < v^{n+1}$$

otherwise

$$\frac{1 - v^{n+1}}{1 - v} = A_{\overline{n}|i} < A_x$$

This will be clearly true if we start from the inverse inequality, i.e. if it is true that

$$A_{\overline{n}|i} \geq A_x$$



then it follows from the fact that

$$A_x \geq v^{h+1}$$

and vice versa!

In the case of assuming gross premium according to the above the back end guarantee time is only chosen by the insured person if the

$$a_{\overline{h+1}|} \leq \frac{1 + \lambda^g}{1 + \lambda^h} \cdot a_x$$

inequality is met. In this case the client must maximize the

$\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{(a_x + A_x - a_g) \cdot (1 + \lambda^h)}$ quotient. The question is whether this is a function increasing or decreasing by g ?

We know that $\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{a_x + A_x - a_g}$ is increasing by g , it is true that $a_{\overline{h+1}|} < a_x$.

Therefore obviously the $\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{(a_x + A_x - a_g) \cdot (1 + \lambda^h)}$ will also be increasing by g , if it is

true, that $\frac{1 + \lambda^g}{1 + \lambda^h} \leq 1$, since in this case it is met that $a_{\overline{h+1}|} < a_x$.

If $\frac{1 + \lambda^g}{1 + \lambda^h} > 1$, then the $\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{(a_x + A_x - a_g) \cdot (1 + \lambda^h)}$ will clearly be increasing by g too, if it is true that $a_{\overline{h+1}|} < a_x$. So the question is, how does the

$\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{(a_x + A_x - a_g) \cdot (1 + \lambda^h)}$, ha $a_x \leq a_{\overline{h+1}|} < \frac{1 + \lambda^g}{1 + \lambda^h} \cdot a_x$ behave?

If we increase g by one, then the numerator increases by $v^{h+1} \cdot v^g$ and the denominator increases by $A_x \cdot v^g \cdot (1 + \lambda^h)$. The question is will the following inequality be true (is when the denominator increases by increasing the g):

$$\frac{a_{\overline{h+1}|} + v^{h+1} \cdot a_g}{(a_x + A_x - a_g) \cdot (1 + \lambda^h)} < \frac{v^{h+1} \cdot v^g}{A_x \cdot v^g \cdot (1 + \lambda^h)} - \frac{v^{h+1}}{A_x \cdot (1 + \lambda^h)}$$

Transforming the equation we receive that it is true if it exists that:

$$a_{\overline{h+1}|} \cdot A_x + v^{h+1} \cdot a_g \cdot A_x < a_x \cdot v^{h+1} + A_x \cdot a_g \cdot v^{h+1}$$

Simplifying it, if:

$$a_{\overline{h+1}|} < \frac{v^{h+1}}{A_x} \cdot a_x$$



Since

$$A_{x:n|g+1} < \frac{1 + \lambda^g}{1 + \lambda^h} \cdot A_x$$

therefore it certainly exists, if it is true that

$$\frac{v^{n+1}}{A_x} \leq \frac{1 + \lambda^g}{1 + \lambda^h}$$

In relation with this inequality we certainly know two things.

On the one hand: $\frac{1 + \lambda^g}{1 + \lambda^h} > 1$

On the other hand we know, that if $A_x \leq A_{x:n|g+1}$, then the consequence of this is, that $v^{n+1} \leq A_x$, i.e. that

$$\frac{v^{n+1}}{A_x} \leq 1$$

The two together precisely means that the $\frac{v^{n+1}}{A_x} \leq \frac{1 + \lambda^g}{1 + \lambda^h}$ exists.

So it is also true, that by increasing g the service-premium coefficient increases, so the client always chooses in the case of the back end guarantee time annuity the biggest one from among the possible warranties.

3.4.3.2. In the case of front end guarantee time annuity

In the case of net premium according to the above said the client only chooses the front end guarantee time annuity, if the $h < g$ inequality is certainly met. In this

case the client maximizes the $\frac{A_g}{A_g + g A_x}$ coefficient. Its reciprocal $\frac{A_g + g A_x}{A_g} = 1 + \frac{g A_x}{A_g}$ function of the g is clearly decreasing, since if the denominator increases the numerator increases, so the original coefficient increases by the increase of g , i.e. the client aim at buying the possible biggest guarantee time from among the front end guarantee time annuities.

In the case of assuming the gross premium it is obvious that if it increases

function to g , then the $\frac{A_g}{(A_g + g A_x) \cdot (1 + \lambda^g)}$ also increases by the increase of g , i.e. in the case of gross premium it will be true, that the client chooses from among the guarantee time annuities the one with the highest guarantee time .



In summary: from the aspect of choice among the possible warranties the size of the loading has no significance, but naturally only if the loading is identical in respect of every guarantee time. If it is different, naturally it is possible, that the smaller guarantee time annuity is the more favorable choice if its loading is adequately smaller than the one of the bigger guarantee time annuities.

3.4.4. The maximum possible rate of loss due to adverse selection

In order to assist the calculation I have created a unisex extinction order based on one of the man-woman mortality tables, where I have weighted the mortality probabilities by the rate of the population of given age. (The selection of the extinction order may have happened among relatively wide boundaries, because I was not interested on the impact of the extinction order itself, accordingly I worked with different kinds of optional extinction order.) Based on this I calculated the net premium of all the possible annuity types with any kind of guarantee time from 1 year to 40 years, and looked at how the difference between the received services and the premium compares to each other at the individuals of different life time. Then I took into account that the client chooses that option where this value is the highest, so I could make an estimate on choice related maximum loss of the provider.

In the model I have developed the mortality tables (between 1990 and 2000) providing the basis for the unisex table can be changed so that in each case I compute the unisex table by weighting the gender distribution of the population of the given year. Between the age of 50

And 70 the age of entry can be selected, the technical interest rate can be changed (though below I mostly use the results received at 0%), and one can also change the number years of guarantee time as a maximum that can be taken into account (between 1 and 40⁴⁴). The results published below were made with the select mortality table of 1990 and with entry age of 62 (in my otherwise the mortality table only minimally changed the results in my experience).

⁴⁴ Respectively – the front-end guarantee time case – maximum to the age of 101 i.e. with an entry age of 62 39 years of guarantee time can be taken into account – this can also be seen at the outcome of the calculations.



If we calculate with 0% technical interest rate and presume that everyone in advance knows the year when he/she dies, and then everyone chooses so that he/she receives the maximum service compared to the payments (either if it received during the lifetime or after the death) then we receive the following results:

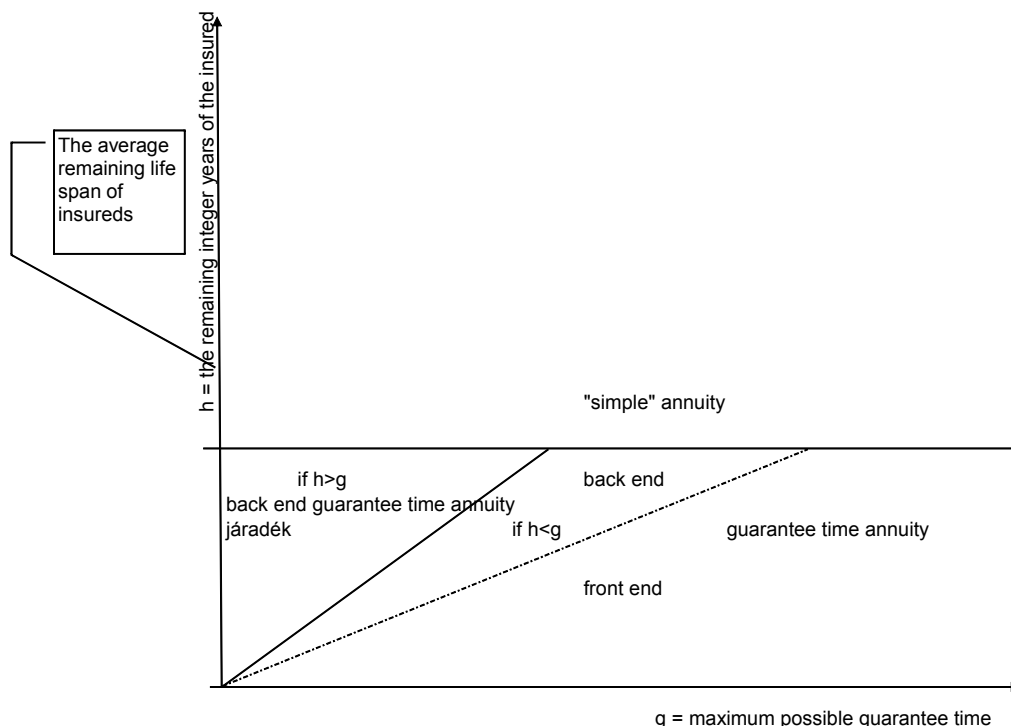
Table 1: The rational choice of clients among the single life annuities

The rational choice of the different clients among the single life annuities, if all the services are maximized													
Age at the time of the contract 62 years													
Mortality table: 1990													
Technical interest rate: 0%													
Age at the time of death	Maximum possible guarantee time												
	1	2	3	4	5	6	7	8	9	10	20	30	40
62	1 back	2 back	3 back	4 back	5 front	6 front	7 front	8 front	9 front	10 front	20 front	30 front	39 front
63	1 back	2 back	3 back	4 back	5 back	6 front	7 front	8 front	9 front	10 front	20 front	30 front	39 front
64	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 front	9 front	10 front	20 front	30 front	39 front
65	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 front	10 front	20 front	30 front	39 front
66	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 front	20 front	30 front	39 front
67	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
68	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
69	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
70	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
71	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
72	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
73	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
74	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 front	30 front	39 front
75	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 back	30 front	39 front
76	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 back	30 front	39 front
77	1 back	2 back	3 back	4 back	5 back	6 back	7 back	8 back	9 back	10 back	20 back	30 front	39 front
78	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
79	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
80	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
81	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
82	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
83	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
84	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
85	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
86	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
87	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
88	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
89	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
90	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
91	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
92	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
93	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
94	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
95	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
96	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
97	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
98	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
99	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple
100	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple	Simple

The results are completely in accordance with the above logical deduction and they may be summarized as below:



Figure 2: The choice of the client among single life annuities, if all the services are maximized



- Those who live shorter than what is expected as remaining lifetime at the age of 62 (this is 15.732 that is it is between the age of 77 and 78) those choose some of the guarantee time annuities with no exception, those who live longer choose annuity without guarantee time without exception.
- The clients do not choose among the possible warranties, in each case they opt for the maximum possible one – though not uniformly front or back end guarantee time annuity.
- From the lower possible warranties first the back end guarantee time annuity is chosen with no exception, from the very high warranties the front end one is chosen with no exception. In the case of transitory possible warranties the front end guarantee time first appears at people with lower remaining life time and with the increasing of the possible guarantee time the remaining lifetime is more and more increasing, when the front end guarantee time annuity is worthy to buy and not the back end guarantee time annuity.



The above results are fundamentally true even if we increase the technical interest rate with the following diversions:

- The higher the technical interest rate, the more increases the individually remaining life time, from when the clients opt for simple annuities
- With the increasing of the technical interest rate the front end annuity is less frequently chosen compared to the back end guarantee time .

For the maximum extent of auto selection function to the maximum possible guarantee time and technical interest rate we receive the following results:

Table 2: The maximum possible rate of the adverse selection

The effect of adverse selection function of the maximum possible guarantee time and the technical interest rate Age at the time of contract 62 years Mortality table: 1990										
Technical interest rate	The maximum possible guarantee time									
	1	2	3	4	5	10	15	20	30	40
0,0%	0,00%	0,00%	0,00%	0,00%	-0,46%	-3,58%	-8,01%	-12,59%	-14,78%	-12,56%
0,5%	-0,26%	-0,49%	-0,72%	-0,93%	-1,56%	-4,72%	-9,09%	-12,61%	-13,93%	-12,38%
1,0%	-0,55%	-1,05%	-1,52%	-1,96%	-2,78%	-6,61%	-10,62%	-13,92%	-15,01%	-13,99%
1,5%	-0,93%	-1,77%	-2,55%	-3,27%	-4,32%	8,93%	-13,31%	-16,05%	-17,27%	-17,00%
2,0%	-1,37%	-2,61%	-3,74%	-4,77%	-6,08%	-11,19%	-15,82%	-18,84%	-20,56%	-20,76%
2,5%	-1,86%	-3,53%	-5,04%	-6,40%	-7,99%	-14,08%	-18,79%	-22,06%	-24,16%	-25,06%
3,0%	-2,45%	-4,63%	-6,58%	-8,33%	-10,23%	-17,41%	-22,62%	-25,88%	-28,54%	-29,88%
3,5%	-3,06%	-5,76%	-8,15%	-10,28%	-12,50%	-20,41%	-26,17%	-29,76%	32,95%	-34,64%
4,0%	-3,57%	-6,70%	-9,47%	-11,93%	-14,13%	-23,27%	-29,48%	-33,07%	-36,72%	-38,70%
4,5%	-4,01%	-7,51%	-10,60%	-13,34%	-15,78%	-25,73%	-32,04%	-36,15%	-40,13%	-42,26%
5,0%	-4,44%	-8,31%	-11,71%	-14,72%	-17,40%	-28,11%	-34,80%	-38,89%	-43,20%	-45,49%

The explanation of the figure: if the insurer calculates the premium of the different single annuities with not taking into consideration the effect of auto selection due to the option, the clients choose the best modality for themselves, then the premium collected by the insurer will be that much lower than the service paid. So e.g.: with 10 years maximum possible guarantee time and 0% technical interest rate the insurer collects 3,58% less premium as a result of the auto selection, than what it pays as a service. (Naturally this calculation does not include that possible loss of the insurer that the actual mortality of the insured does not correspond to the calculated one.)

The results can be evaluated as follows:

- With 0% interest rate and low guarantee time (4-5 years) the choice practically does not have auto selection impact.



- With 0% interest rate the higher the possible guarantee time, the auto selection impact is the higher for a period of time (in the table it is -14,78% with 30 years, i.e. that less will be the amount of total premium collected than the payments), later it decreases at a lower rate
- With the other technical interest rates a similar trend prevails, the higher the technical interest rate the effect of auto selection reaches its maximum at the higher guarantee time.
- The higher the technical interest rate the stronger the effect of auto selection. Altogether the interest rate has a strong impact on the auto selection.

3.4.5. Possible strategies to diminish the loss due to adverse selection

It is reasonable to change (to refine in some cases) the Hungarian regulation in force on the possible annuity parameters as follows:

- To rule that the several life annuity can only be annuity for two lives (otherwise the segmentation strategy may be applied in the case of two lives annuity)
- Because of the later calculation and the consistency in time it is reasonable to rule that the annuity is monthly and the number months is 12
- From the calculations it can be seen that up to the possible maximum 5 years guarantee time with 0% technical interest rate the effect of auto selection due to the choice is minimal (-0,46%, what due to the extreme presumptions in reality is only a fragment of that), therefore depending on the magnitude of the technical interest rate there are two options:
 - If it is bigger than 0%, the regulation generally abolishes the possibility of applying the guarantee time,
 - If it is 0%, the guarantee time is either abolished or maximized in 5 years
- We can see from the calculations that the client making a rational choice either does not opt for a guarantee time, or chooses the maximum one from among the possible ones. The consequence is that there is no reason to offer different warranties.



- The possible guarantee time also depends on the price of the guarantee time. The below figures show that compared to the annuities without guarantee time how much more the different warranties cost (calculated with 0% technical interest rate and with unisex mortality table). It can be seen that the guarantee time at the beginning is a fairly cheap thing, though at the end it is rather expensive:

Table 3: Relative premium of front end warranties compared to annuities without guarantee time

Relative loading on premium of front end warranties compared to annuities without guarantee time								
Entry age	Length of Guarantee time							
	5	10	15	20	25	30	35	40
50	0%	2%	5%	11%	19%	30%	45%	62%
55	1%	4%	9%	18%	30%	47%	68%	90%
60	1%	6%	15%	28%	48%	72%	100%	128%
62	1%	7%	18%	35%	58%	85%	116%	
65	2%	10%	25%	47%	76%	109%	144%	
70	4%	18%	43%	79%	121%	164%		

Table 4: Relative premium of back end warranties compared to annuity without guarantee time

Relative premium of back end warranties compared to annuity without guarantee time												
Entry age	Length of guarantee time											
	1	2	3	4	5	10	15	20	25	30	35	40
50	4%	8%	12%	16%	20%	40%	61%	81%	101%	121%	141%	162%
55	5%	10%	14%	19%	24%	48%	71%	95%	119%	143%	167%	190%
60	6%	11%	17%	23%	28%	57%	85%	114%	142%	171%	199%	228%
62	6%	12%	18%	25%	31%	62%	92%	123%	154%	185%	216%	246%
65	7%	14%	21%	28%	35%	70%	105%	140%	174%	209%	244%	279%
70	9%	18%	26%	35%	44%	88%	132%	176%	220%	264%	309%	353%

- Based on the above it is reasonable that the only guarantee time permitted by the law is 5 years in the front end and 1 year in the back end guarantee time version.
- The following figure may also be interesting where it indicates that with the same presumption (1990 mortality table and 0% technical interest rate, that what percentage of the net premium the insured person gets back as guaranteed service (i.e. as service during the guarantee time – irrespective to the actual life time):



**Table 5: The insured person is guaranteed to get back this portion of the premium –
Front end guarantee time annuity**

Guaranteed portion of the premium what the insured person gets back – Front end guarantee time annuity												
Age at entry	Length of guarantee time											
	1	2	3	4	5	10	15	20	25	30	35	40
50	4%	8%	12%	16%	20%	40%	57%	73%	85%	93%	98%	
55	5%	10%	14%	19%	24%	46%	66%	81%	91%	97%	99%	
60	6%	11%	17%	23%	28%	54%	75%	89%	96%	99%	100%	
62	6%	12%	18%	24%	30%	58%	78%	91%	98%	100%	100%	
65	7%	14%	21%	28%	34%	64%	84%	95%	99%	100%	100%	
70	9%	18%	26%	35%	43%	75%	92%	99%	100%	100%		

**Table 6 : the insured person is guaranteed to get back this portion of the premium –
Back end guarantee time annuity**

Guaranteed portion of the premium what the insured person gets back – Back end guarantee time annuity												
Age at entry	Length of guarantee time											
	1	2	3	4	5	10	15	20	25	30	35	40
50	8%	11%	14%	17%	20%	32%	40%	47%	52%	57%	60%	63%
55	9%	13%	17%	20%	23%	35%	44%	51%	57%	61%	64%	67%
60	11%	15%	19%	23%	27%	40%	49%	56%	61%	65%	69%	71%
62	12%	16%	21%	25%	28%	42%	51%	58%	63%	67%	70%	73%
65	13%	18%	23%	27%	31%	45%	55%	61%	66%	70%	73%	75%
70	16%	22%	28%	33%	37%	52%	61%	67%	72%	75%	78%	80%

- If the above proposals on the restriction of the guarantee time are included in the regulation, these manage the problem of auto selection due to the choice to the extent, that among these circumstances it will be a minimum what is worthy to deal with. However if the decision makers create such rules, that the guarantee time may be much longer and/or the different providers may tighten their annuity-offers (e.g.: so that they offer only back end guarantee time annuity, and by this they attract the not too healthy men who are of the best risk from annuity aspect) then a separate auto selection management may become necessary as a result of the choice. Namely in this case the general increase of premium level might become necessary to compensate the auto selection. As for the rate of this the figure indicating the effect of auto selection may give orientation.

3.5. DIRECTED SELECTION

If the insured people may choose from among different types of annuities at a provider those with shortest life expectancy choose front end guarantee time annuity,



those of longer than that life expectancy but of shorter than average life opt for the back end guarantee time annuity, but those of longer life expectancy than the average choose the simple annuity. If we calculate all the three types of annuities with identical mortality table, then the result of this selection will be, that the insurer makes profit on the guarantee time annuities, and loss on the simple annuities and in the best case its profit and loss offsets each other.

The possibility of solving the problem can also be that the annuities are calculated with different mortality tables so that it is adjusted to the mortality of those who, on rational considerations, voluntarily opt for that. If this can be done, the insurers might take the advantage of selection instead of struggling against it, by this they would get a risk community selected from risk aspect, where the clients themselves voluntarily, based on rational considerations select the risk community which they want to join. Insurers may potentially support the clients so that they present the client which annuity is advantageous for them and should somebody be uncertain about how long he would live, insurers may propose them an adequate health test, which can help them to assess the length of life in their case. After this the insurer lets them choose. By this the risk assessment can be spared, since by the help of precise calculations, trained advisers it is clearly published what is advantageous for whom. This also helps to avoid problems resulting from concealing the data from assessment.

The question really is whether this is possible at all, would this effort of solution not lead to contradiction. Namely if it turns out that this is not the way to go, precisely the opposite must be done. It is possible for the insurer to decrease the risk for itself by applying the above described self selection of annuitants, so that the annuities provided are drastically diminished compared to the possible annuities. If for example a provider only gives guarantee time annuity, it means that its risk is drastically diminished since:

1. guarantee time annuity can a priori be provided with smaller risk than the one without guarantee time



2. by this it attracts the short life (primarily male) insured, and makes itself less attractive for the long life expectancy (mainly female) insured; by this its risk community is indirectly selected.

Because of the consequence of the item 2, it must be seriously considered whether this possibility is permitted. It is better if in this case the law excludes this possibility, and provides that an annuity provider insurer is obliged to provide all the annuities granted by the law.

As I present below this solution is not possible, so the targeted selection does not work. I conduct the examination as I did above for the option of simple, front end and back end guarantee time annuity.

The question whether auto selection due to the choice may be eliminated by using adequate mortality tables can be restricted to the question whether auto selection may be eliminated by increasing the premium, decreasing the premium respectively (as the client takes into account the different mortality tables only via the premium).

The answer is not clear, since if we change the mortality table, the premium of the annuity changes too, along with the composition of those who opt for that annuity versus the others. The question is whether this change converges to such a state that every type of annuity remains in the market for an adequate range of clients or in reality certain types of annuities loose ground.

In the case of 3 lives annuity this question can further be simplified at the examination: can the auto selection loss suffered due to the choice of simple annuity be eliminated by increasing the premium of the simple annuity or by decreasing the premium of guarantee time annuities. From this point of view it is of secondary importance if auto selection presents itself if the insured may only choose between two types of guarantee time annuities and if the answer is yes whether the auto selection can be eliminated by increasing or decreasing the premium. As this question is not a practical one, I do not give an answer.



3.5.1. Adverse selection can be eliminated by increasing the premium of the simple annuity

3.5.1.1. In the case of the simple and of the guarantee time annuity

In the **case of net premium** those people choose simple guarantee time, in whose case it is true that

$\ddot{a}_{\overline{N}|1} \geq \ddot{a}_x$, i.e. those people did so, who would certainly get as much in service as much premium they pay, since in this case, and only in this case - $\frac{\ddot{a}_{\overline{N}|1}}{\ddot{a}_x} \geq \frac{\ddot{a}_g + v^g \cdot \ddot{a}_{\overline{N}|1}}{\ddot{a}_g + v^g \cdot \ddot{a}_x}$ - will be true. If we increase the price of the simple annuity

$$\ddot{a}_x$$

from \ddot{a}_x to \ddot{a}_x' , while in the meantime we leave unchanged the price of the back end guarantee time annuity (i.e. in the case of the simple annuity we assume higher remaining life), then it is obvious, that those who so far chose back end guarantee time annuity (i.e. in whose case if $\ddot{a}_{\overline{N}|1} < \ddot{a}_x$ it was true), those would keep on buying that, because they have no reason of whatsoever to change.

However in the case of those at who $\ddot{a}_x \leq \ddot{a}_{\overline{N}|1} < \ddot{a}_x'$ is met, $\frac{\ddot{a}_{\overline{N}|1}}{\ddot{a}_x} < \frac{\ddot{a}_g + v^g \cdot \ddot{a}_{\overline{N}|1}}{\ddot{a}_g + v^g \cdot \ddot{a}_x}$ will always be true, as it is true

that $\frac{\ddot{a}_{\overline{N}|1}}{\ddot{a}_x} \leq 1 < \frac{\ddot{a}_g + v^g \cdot \ddot{a}_{\overline{N}|1}}{\ddot{a}_g + v^g \cdot \ddot{a}_x}$. So a part of the clients, who chose simple annuity with a lower premium would shift to the back end guarantee time annuity now. In addition all those will be the clients who change, in whose case the received service is smaller than the premium to be paid for the simple annuity, so only those people will keep on buying simply annuity, who receive bigger service than the premium of the simple annuity.

Based on the above we can ascertain that if we increase the price of the single annuity we exclude those clients from buying simple annuity who would have chosen that at a lower price, but the new, higher prices exceed the anticipated service. Only those clients choose the simple annuity with the new price, at whom the anticipated



service will further be higher than the price and the others convert to buying back end guarantee time annuity. By this the simple annuity further remains loss making. So by raising the price of the simple annuity we can not achieve that the simple annuity becomes profitable for the provider, this way the simple annuity loses ground in the market.

Presuming gross premium does not significantly change the above said, the logic of net premium can be applied here too, so auto selection can be eliminated by increasing the premium.

3.5.1.2. In the case of simple and front end guarantee time annuities

In the net premium case based on the above the borderline between the simple and the front end guarantee time annuity is the
$$\frac{a_{\overline{n}|i}}{1+i} < \frac{a_{\overline{n}|i}}{1+i} + \frac{a_x}{1+i}$$
 inequality (when $h < g$ will obviously be true as well, respectively the coherence $\frac{a_{\overline{n}|i}}{1+i} < a_x$ is also valid). If the inequality exists, then the client chooses front end guarantee time if not simple annuity.

It is obvious that the best clients of the simple annuity would choose the front end guarantee time annuity, therefore the simple annuity will become loss making however not to the extent as we could see it in the case of the simple and back end guarantee time, because such people also buy simple annuity in whose case the received service will be smaller than the premium paid. The question is whether the profitability of the simple annuity may be restored by increasing the premium of the

$$a_x$$

simple annuity that is if we increase the premium from a_x to $a_x + \frac{a_{\overline{n}|i}}{1+i}$.

In the context of increasing the premium it is immediately raised whether there is some cap on it. It is completely obvious that the premium of the simple annuity can be increased above the premium of the front end annuity, because there would be no argument in favor of buying simple annuity instead of the front end guarantee time annuity. In fact we may be a little bit more severe as well: the premium of the simple annuity must be lower than the premium of the guarantee time annuity. As this is the



single reason for which the client buys the annuity offering lower service from among two annuities of identical premium.

If the premium of the simple annuity were raised to the level of the one with front end guarantee time, and if we were to suppose contrary to the above said that only those individuals buy front end guarantee time annuity in whose case the guarantee time really guarantees something based on their remaining lifetime, then we may say that in the case of a provider offering two (i.e.: simple and front end guarantee time) annuities the auto selection could really be eliminated. Among these conditions the provider actually sells only an annuity, one with front end guarantee time and in the case of an annuity the problem of auto selection due to the choice can not emerge, at least not on the level of the provider. So the insurer collects as much premium from the entire risk community as much it pays to them. However the clients – also based on the above – are divided so among the two annuities of formally different types that individuals whose lifetime remains below the guarantee time choose the guarantee time annuity, the rest of the people opt for the simple annuity. Among such circumstances people choosing guarantee time annuity obviously pay more as premium than what they receive as service therefore those opting for simple annuity obviously pay less if the entire risk community is in balance in respect of premium and service. So the loss due to the simple annuity can not be eliminated by raising the premium.

Presuming **gross premiums** does not change in merit the above logic.

3.5.2. By decreasing the premium of guarantee time annuities

In the case of **net premium** the question is whether the range of simple annuity buyers grows in response to the reverse strategy i.e. in response to reversing the premium of whichever (front or back end) guarantee time annuity. Obviously not, because those individuals for whom it was worth buying guarantee time annuity it is even more so. What's more a part of simple annuity buyers would shift to guarantee time annuity buyers – those who have the shortest remaining lifetime from among the simple annuity buyers. These were though the most favorable clients – from simple annuity aspect, so this way the simple annuity may become even more loss making, so the related loss can not be eliminated.



Presuming the gross premium does not change the above logic in merit.

3.5.3. Summary

In summary: if we presume that the clients know their remaining lifetime and all of them maximize the total received service compared to the premium, even in this case it is impossible to achieve that the single annuity is profitable in itself, neither by increasing the premium of single annuity nor by decreasing the premium of some of the guarantee time annuities, maximum the single annuity loses ground in the market. In other words it means that it is impossible to prepare such selection tables that reflect the mortality of individuals opting for different annuities.

3.6. SELECTION IN THE LITERATURE

The technical literature and the legacy experience suggest that the mortality of annuitants is lower than the entire population (as I have already referred to some such cases in respect of differentiation). The English literature calls this phenomenon as “adverse selection” ((I call it auto selection), and a large portion of the technical literature this covers the otherwise much larger notion. According to Bein-Bogyó-Havas [1907] (page 162) “The experience of the annuity providing companies suggests that the mortality of annuity insured individuals is lower as an average than what is presented in the mortality tables completed from the ordinary mortality insurances. The explanation of this is that the annuitants regularly live among better financial conditions, therefore they are less exposed to the influences of the struggle for life what deteriorates life expectancy. Recently many associations prepared separate tables based on their own experiences; they calculate the deposits needed for annuity provision on this basis.” Bein-Bogyó-Havas did not yet use the word selection for this, but later it has become a common place. In relation with annuities selection is referred to primarily in this context, so individuals voluntarily opting for annuity are severely selected compared to the entire population and this causes severe damage for the insurers, so this selection is “adverse” in the English literature.

According to Hylands-Gray [1902] the annuitants may conduct a kind of self selection, so an individual in bad health condition would rarely (voluntarily) conclude annuity insurance. In our view not only their mortality is more favorable than the one



of the entire population, but also the signals of the temporary initial selection can also be observed (such phenomenon can also be observed at other life insurances, if for nothing else because of the risk assessment of the insurer, what is not characteristic for the annuities).

Interestingly the thick volume - of Booth-Chadburn-Cooper-Haberman-James [1999] , what was meant to be the summary of modern actuarial science – does not include neither the expression adverse nor the term selection, and moral hazard is not specifically elaborated either. Though they also mention that in the case of annuities (page 226-227) risk assessment is not necessary since the individuals applying for annuity are “self-selected”, therefore their expected lifetime is higher than the one of the entire population.

Relatively less reference is made to the selection impact of choice among the different annuity types, but this can also be found. E.g.: Watson Wyatt Partners [2002] point out that in the case of non indexed (unchanged) annuities that this is definitely advantageous for those expecting short life.

Cardinale-Finlater-Orszag [2002] says that the literature on annuities focuses on the individuals' costs and auto selection. The most complete and most quoted summary of the theme is given by Amy Finkelstein and James Poterba in two studies (Finkelstein-Poterba [2002] and [2004]. The Finkelstein-Poterba [2002] study does not only examine the selection resulting from the voluntary annuity choice typically discussed in the literature, but – as they say - the selection resulting from expansion of the DC systems as well. They conducted examination not only in the – in their opinion small – US annuity market, but the bigger UK market too. In their view the UK market is good because not only the voluntary but the mandatory annuity market exist, therefore the evolution of life time among the annuitants may be examined in both of the markets.

Finkelstein – Poterba [2002] finds the roots of auto selection in the information asymmetry between the clients and the insurer. The client chooses on the basis of better information than the insurer whether to buy annuity or not and if yes, what annuity to buy. The study distinguishes two types of selection, the one at the time of entry and the selection within the market. Individuals with long life expectancy have



better incentive for annuity purchase (entry selection) and similarly to choose the type of annuity. They prefer the unchanged real value (i.e. indexed to inflation) called “backloaded” annuities, contrary to the nominally fixed annuities. Contrasted with these individuals with short life expectancy buy guaranteed annuities (with either guarantee time or other payback guarantee), what allows them for legacy. Auto selection naturally is only the case if the information is really asymmetric; otherwise the insurer adjusts the price to the individual risk. In our view the insurer may take into account the mortality of those, who buy that type of insurance (and of which I proved that finally it is not possible!). Active and passive selection is distinguished. In the first case someone decides about the purchase of annuity aware of his mortality, in the second case though he has no information about his mortality, but makes a decision about the purchase with regard to a factor in correlation to that – e.g. his wealth. Both cases lead to impacts similar to selection.

Finkelstein and Poterba – contrary to me – examine auto selection taking place in an existing annuity market (in the lack of such market I could not examine that). They are with the strong presumption that the insurers know the rate of selection and this is reflected in the premium of annuities too. This of course is a rational presumption – in a competitive annuity market of long history, presuming old, experienced annuity providers - since in such markets the providers might have empirically come to the adequate pricing, and the strong safety overpricing is excluded by the competition. The method of their examination is that they look at the diversion from the pricing according to the one of a selected “etalon” mortality table of different annuity provisions. This diversion is called the “money’s worth” of the annuity, what is the proportion of the discounted value to be paid according to the “etalon”. This is one if the premium of the annuity is actuarially correct. The costs, anti selection, etc. bring it below one. If we use select mortality table, it is even more below one, but even if we calculate with the mortality table of annuitants we will be below one because of the costs.

The authors found proof in the annuity premiums for both of the examined selections. In respect of the selection within the market they identified that the annuity money worth almost always decreases with the progress of the age, as well



with the increase of the magnitude of the premium. An other experience was that the more the guarantee time increases, the bigger is the money worth (so - as it was expected – the presumptions that guarantee time annuities are mainly purchased by individuals of shorter life time).

They have two sorts of proof for the entry selection. First, that the annuity mortality tables completed by the “Institute of Actuaries” are compared to the select mortality tables. According to this it is obvious that survival probabilities are the biggest in the voluntary annuity market, next in the mandatory annuity market, finally they are the smallest in the entire population. The second proof is exactly this, that in the case of any annuity products the selection is half of one of the voluntary market. The consequence of this for us is that making annuity mandatory would be quite useful among others in order to diminish selection.

Finkelstein and Poterba repeated their examination two years later too (Finkelstein- Poterba [2004]), when they had access to the data base of the United Kingdom insurer and they could test their presumption on that. They experienced that auto selection according to the amount of the premium is very small – though other studies generally examine auto selection according to this. However strong selection was found according to other parameters. Strong proof was found that people with longer life time buy more back-loaded (e.g.: indexed, possibly indexed to inflation plus) annuities both in the voluntary and the mandatory market. The guarantee time annuities are rather bought by individuals of shorter life time, though the difference in lifetime of buyers guarantee time and back-loaded annuities in the mandatory market was not statistically significant! On the mandatory market those who get bigger annuity are of longer life time but it is not the case on the voluntary market! The explanation may be that a priori the very rich are present on the voluntary market, so there is no big mortality variation here already. In the voluntary market the mortality differences among the guarantee time and non guarantee time annuitants are only a little less, than between men and women at the same place, however the differences according to the initial amount are small.

In relation to the evaluation of the results they call the attention that their results may also be interpreted so, that those who buy annuity change their behavior



so that they can live longer – that is the results may be explained by moral risk as well! The other possible alternate explanation is that here we do not have asymmetric information and competitive market, but we have symmetric information and non competitive market, what may lead to similar results. However this probability is not seen as possible. The third possible explanation is that individuals have diverse preferences, they do not belong to the same risk type and none of them are monitored totally by the insurers (as I put it depends on what the target function is! - J.B.) such items of preference may be: interest rate, risk avoidance, desire for legacy and they may correlate with the risk type.

The literature widely knows and quotes the solution to diminish auto selection by making annuity purchase mandatory) naturally this may only be the case in mandatory systems or in those DC systems where there is a tax incentive. I have just quoted Finkelstein-Poterba [2002], but this idea already appeared at James (edit) [1994] and Hylands-Gray [1992]. They also make the remark that in such cases individuals of bad health conditions probably opt for the longest guaranteed payment period; therefore adverse selection for the insurer may also occur here as well, though in it is doubtful that this would be of significant magnitude. Therefore there was a debate about rendering annuity mandatory in the United Kingdom (Wadsworth [2002]). The idea is included in the OECD recommendations (Antolin [2008a], respectively OECD [2008a], however it mentions that mandating is deemed to be a rough solution, and here mandating is made default, what leads to almost identical result as soft compulsion according to experience. According to him this is really effective is the financial education is enforced parallel for the sake of promoting annuities.

In the technical literature the idea of conducted selection (and its denial) appears too, as I have described. According to James (edit.) [1994] the redistribution as the result of the uniform premium can be mitigated so that insured of different risk are offered different type of contracts and they may make a choice from among those. According to them, if every type of contract is priced so that they are self-supporting this diminishes the problem of auto selection and non desired redistribution.

The issue of selection has not been systematically processed – apart from me – in the Hungarian literature, rather scattered ideas can be found. For example



Michaletzky [1999] raises implicitly, and in specific cases the problem of auto selection (page 100): “it may happen that it is more advantageous for the fund too if the insured individuals acquire the service from some of the insurance companies, by this weakening the risk community of the fund.” He does not prove this opinion; therefore it may only be regarded as a paranoid fear of the insurer. Nevertheless the motive that insurers are selected voluntarily by the risk people appears at György Németh too (Németh [2006]). Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] only mention the theme of “antiselection” due to the option, Stahl does not even come back to this proposal later in 2005. This is raised in relation to changing the provider during the maturity, what he already projected in the law, and he credited this with a quality of antiselection: “A few provisions of the law suggest that according to the intention of the legislator the movement among the providers is allowed in the course of annuity provision as well. This may have antiselection effects as well. Identical effect may have the possibility of option from among different annuities, as well that prior to commencing the payment the amount on the account may be inherited in the form of lump sum payment. “

Later Miklós Arató disputing with Stahl identified two important selection impacts of the private pension annuity system, and also suggested the solution (Arató [2006a]). Partly he highlights that though the membership in mandatory private pension system started with voluntary entry so annuity will have to be paid on long term for a select stock, which will probably have longer life time than the entire population. Therefore it would be important to find out and project the actual mortality of the membership as soon as possible. On the other hand he recognizes that page 272) “...the high number of probable types of annuities inhibits hazard as well. After a while people find their way and choose the policy relevant to their health condition, for example the sick people would choose guarantee time annuity with the possible longest guarantee time”.

Kolos Ágoston and Erzsébet Kovács highlight the auto selection impact of mandating what they also see manifesting in the option among annuities (Ágoston – Kovács [2007]). According to them when applying the uniform (unisex) mortality table the mortality of men and women is averaged in proportion to the number. This does not mean perfect solution because the individual decisions – for example men buy two



lives annuities, women buy one life annuity – turns over the pre set proportions”. They are also concerned that due to the difference of mortality between genders “unpredictable migration may start among funds, since everyone tries to get into a fund where there are more young men. Probably funds try to keep women apart from funds, for example by setting up closed “professional” funds like the “miners’ fund”.

The technical literature relatively little discusses the selection impact of choosing the otherwise mandatory pension annuity at its start, though references are made to this as a well known phenomenon. For example Winkler-Mattar [1999] proposes the insurers that for the sake of decreasing the risk postponement of the start should be allowed to only a certain extent. In respect of this subject it is worthy to mention that András Simonovits devoted a whole series of studies to this issue in the past years (the first two were Simonovits [2001] and Simonovits [2002]). He is critical about the free option at the start of the annuity, and about the so called “actuarially correct” setting of annuity, but his criticism is mainly targeted to these efforts of Pillar I (prominently to the NDC solutions). Though his statements could be generally applied to the DC type of Pillar II. annuities, still I do not do that because I have serious problems with Simonovits’s approach, I plan to dedicate a separate study to this. In any case I also discuss the phenomenon and give proposal to the solution as well, nevertheless it is worthy to mention that I have completely opposite view about the problem – as a result of the differences in our approach: my concern is not that individuals with high life expectancy would convert to annuity too late, but that individuals with short life expectancy would not convert to annuity at all. This is possible because of the significant difference between Pillar I. and II. that in Pillar II – contrary to Pillar I. examined by Simonovits – there is not possibility for legacy, and the motive for legacy is missing, therefore quite rightly Simonovits does not examine that.



4. MODEL OPTIONS FOR MANDATORY OLD AGE PENSION BENEFITS

Above I have reviewed the major annuity related problems, the possibilities of managing these problems. In the course of discussing them it was clear that not every item can be voluntarily combined with other items, certain items presuppose other ones, while others exclude each others, and there are also neutral items. Therefore the legislator when developing the annuity system may choose among the different annuity systems or models. However before starting to analyze the possibilities the question is raised, whether it is a must to choose a model, and whether it is not sufficient to let providers and clients to offer, respectively choose such annuity what they want. In my view this is not the way to in the case of mandatory annuities, but the reasons are worthy to be examined.

4.1. THE MODEL OF THE LACK OF MODEL – ENTIRELY FREE ANNUITY SERVICE AS A POSSIBILITY?

Right at the beginning it must be clarified that the notion “entirely free” annuity service is not quite unequivocal, since the annuity market operating with small restrictions is generally regarded as “entirely” free in practice. Consequently the “entire” freedom is also scalable. At one end of the scale is the annuity market operating almost without any rule, at the other end there is the “free” annuity market operating with several, but very general rules.

The annuity market operating almost without any rule can certainly not be logically linked to the system of “mandatory” annuity, the basis of which is that the state does not force citizens to spare continuously and targeted a defined portion of their income during their active carrier for their old age consumption. This force is based on the presumption that the vast majority of citizens is of short sight and undisciplined, and prefers current consumption contrary to old age safety, and this is the reason why a state without obligation would in the end be confronted with a large low income old age population finally relying on social benefit. If this is what we presume of the citizens, if this is the reason for obliging them to accumulate, then this presumption is not in accordance with the regulation that permits them to do whatever they want with their money at the time of retirement, because it can be



presumed that even at that time the majority would spend it in a short sighted way, so the original problem gets reproduced. In other terms the entirely free annuity provision, in reality is the mandatory labor pension, namely the logical system in the lack of mandatory pension savings. Such pension system is also possible, so if there is no central pension system at all, or if there is a basic pension that provides only a minimal service for everyone, there are international examples for this, and it has advocates as well. The detailed analysis of this system is outside the scope of this study as this would need the description of the possible methods of using the mandatory accumulated assets as well.

So for the mandatory savings in the case of “entirely free” annuity service there still is a need for at least one rule: the mandatory savings for pension (or a mandatory fixed minimum portion of that) is obligatory to convert for annuity.

Naturally this rule in itself is not sufficient, since the meaning of the word “annuity) – as I have indicated – is wide enough to include for example the life annuities and annuity certain too. Since the pensioner is more and dependent on the pension income as his age progresses, therefore only the life long life annuity is adequate for mandatory annuity (e.g.: the temporary life annuity is not adequate!). Consequently this rule is worthy to further develop: the mandatory pension savings (or a mandatory fixed minimum portion of that) is obligatory to convert life long (but not necessarily for an immediately starting) life annuity.

Yes, but if there exists such an absolutely minimum regulation, then it is almost certain, that the legislator gets into a sort of “legislature spiral”, so it becomes unavoidable to further elaborate on what is meant by “life long life annuity”. This is due to the fact that if the providers (serving further on the short term interest of short sighted consumers) try to pay out the saving in lump some, or the possible biggest portion of that in the shortest possible time, so to circumvent the goal of the regulation by sticking to it formally. This is possible e.g.: by paying out the life long annuity in decreasing amounts, for example at each payment they pay out 50% of the provision to the client until the end of the life (though this very quickly decreases the “life annuity” to a nominal amount). So it is useful to add, that the life annuity must not be of decreasing amount, etc.



The state takes implicit responsibility by mandating, namely it may not let irresponsible providers appear in the market who steal the clients' money, or later are unable to pay the promised service. The state is forced by this to restrict the range of possible providers, to supervise their activities and by setting the capital adequacy requirement make them able to extend the undertaken service.

Probably the state can not avoid taking a stand in such questions as along what criteria the providers may distinguish among the clients (e.g.: age, gender, occupation, domicile, size of the assets, etc.). In the European Union for example it is forbidden to distinguish among genders at the moment – however this rule is disputable, one must take it into account.

It is also probable that the state faces that it must have its say about the indexing of the annuities: is it compulsory or not, and if yes, that is the basis (inflation rate, wage index, etc.)? If the state provides some sort of obligation in this regard, by this it launches a new spiral of legislation: can the longevity risk be devolved to the client or not; how can be avoided (by new rules) that the provider gets into difficult financial situation as a result of the obligation for which it is not the provider but the state is responsible, etc.

The justification for the free annuity service may be of twofold, a “practical” and an “ideological”. According to the practical justification the legislature work of the state may be saved by letting the market develop the annuity service system. The “ideological” justification says that the competing providers give the best possible result for the client, so they would provide the possible most adequate at the possible lowest price for the client.

Serious counter arguments can be raised against both justifications. All the above counter arguments contradict the “practical” justification, because if there is state obligation in the pension system, then the state inevitably gets into a spiral of legislation. The argument against the “ideological” justification is the Hungarian (and probably many other) life insurance market, where expensive products not meeting the clients' needs have been offered for a long time by the competing providers. The reason for that is obvious and it would clearly work similarly in the “free annuity market” too: in the case of free annuity service the providers would make efforts to



account for the larger cost in the premium of the product. This can be the best achieved if they offer products that are incomparable with the one of the competitor and over value the significance of the difference by the help of promotion advertisements. So the unregulated competition often extends to many parameters in reality mean smaller competition because it actually abuses the modest sense of orientation of the clients. In the case of any intense competition is a competition at a central place, among standardized products, which is symbolized by the operation of the stock exchange. There would be no stock exchange without standardization; therefore the ideological argument builds upon the market vision contrary to the real operation of the market.

Altogether I think that to the mandatory labour pension, the system of mandatory pension savings, the model of entirely free annuity service does not adjust, that may only operate at such a place, where there is not stately mandate for pension savings. Where that exists and still such model is started the expectations towards the state, and the consequent regulatory spiral may easily lead to a state that though the starting point is a completely annuity provision system, but in a short while we have a haphazardly regulated annuity providing system. Therefore it is reasonable to avoid this trap, and to review the expectations towards a mandatory annuity providing system, and it must be inserted into a well considered regulatory system

4.2. THE POSSIBLE ELEMENTS OF THE ANNUITY SYSTEMS AND ITS CONTEXT

In the above two chapters I analyzed how the selection of the different key elements (scope of choice of the insured individuals, applied indexation technique, etc.) determine the other elements of the annuity system, respectively what the resulting problems are and what are the tools for managing them. In summary due to the internal context of the solutions an annuity system can not be built up so, that the above elements are arbitrary attached to each other, those can only be combined with each other in a determined way. The above elements define several interrelated annuity systems, but the decision makers many only opt among these annuity systems; they are not free to mix the different elements.

Below I summarize the above detailed context among the elements, and on these grounds I make an effort to identify the possible annuity systems.



4.2.1. System elements and options resulting from the management of selection problem

4.2.1.1. Selection and its possible management resulting from choosing the provider

There is no selection problem in a completely free annuity service system as the clients' unlimited possibility for option is coupled with the unlimited possibility for distinction (including even the refusal of the potential client as well). However if the regulation restricts the clients' possibility for option, namely (apart from a few parameters) distinction is forbidden for them, then the adequate rule on the part of the clients is that the regulation does not assure them possibility to choose the provider. If the client still does that, the loss must be spread among the providers.

So the possibilities are as follows:

Selection problems due to choosing the provider		Distinction among the insured individuals	
		Unlimited possibility	Limited
Insured individuals from among the providers	May not choose	No selection problem	No selection problem
	May choose	No selection problem	Selection problems!

Hereinafter I do not analyze the possibility that the distinction among the insured individuals is unlimitedly possible⁴⁵, as according to the above said this possibility does not exist in Hungary (and probably neither in the majority of the countries). Though if distinction among insured individuals is limited, the question arises, whether the clients may choose the provider or in other words: are there competing providers in the market?

There is no selection problem, if there are no competing providers in the market.

This may happen in two cases:

1. there a single, central provider
2. there are several providers, but they are closed, and it is clearly defined which client belongs to which provider.

Though if there are competing providers in the market, then the selection problem due to the prohibition of distinction may be equitably managed in two different ways:



1. pool, by the help of obligatory, centrally organized pool
2. by dividing the product(s) into two parts, and by transferring the risky part to an other provider.

Equitable means that theoretically the regulation may transfer the management of problems that are generated by the regulation itself to the provider, so the legislator may say, that the negative consequence of the regulation should be either covered by the from the guarantee time capital, or if the provider can may account it on the clients. Naturally the legislator may allow this latter solution only to a certain extent, so if at the very beginning the problem is not managed the legislator may get into a spiral of regulation similarly to what was described above, and the result may be a scrappy regulation without clear incentives, or it may lead to binding the providers so that they leave the market. So furthermore I do not raise this possibility in either of the solutions.

There may be two types of pool:

- a) minimal (premium equalizing mechanism). This may be applied if the indexing is based upon the refund of excess return, and via that the mortality loss (partly or wholly) is transferred to the clients
- b) maximal, when during the entire maturity the annual mortality results are spread.

If the annuity is split in time into a temporary and a deferred annuity part, then the selection risk is not identical in the two parts. If the duration of the temporary annuity (so the deferral of the deferred annuity) is small, the vast majority of the selection risk is charged at the deferred annuity. The proportion between the two may be changed by increasing the duration of the temporary annuity. In such a case the selection risk on the deferred part decreases. However this can totally be eliminated from the temporary part, if instead of temporary annuity that is made to be annuity certain (or if we make it to be a scheduled money withdrawal, what is identical with that from this aspect) Naturally the price of it is that the annuity unit would be somewhat smaller, the longer the temporary part the more so.

⁴⁵ A regulation based upon such a principle will certainly work with much less restriction, so that can be easily created based on this present study.



So the by the scheduled money withdrawal + deferred annuity facility, where the deferred annuity is transferred to other provider allows the elimination of selection risk for the period of time of the scheduled annuity. The deferred annuity is basically transferred to two types of organizations:

1. to a central provider
2. to a well-capitalized market player

So we get the following possibilities:

1. central provider
2. closed providers
3. competing providers
 - a. organized into a mandatory pool, the entire annuity is paid by the different providers
 - i. premium equalizing mechanism (plus transferring the mortality profit via indexing to the clients)
 - ii. maximal pool
 - b. competing providers may only pay temporary annuity and the deferred part of the life annuity is transferred to a well-capitalized market player
 - c. the same as above, only the deferred part is transferred to a central provider.

From among the above the possibility for a closed provider exists only before the starting up of a private pension system, therefore it is not feasible among the Hungarian circumstances, so I do not elaborate on it any longer. However in this case the statements concerning the central providers are valid, since the closed provider performs its activity as a central provider from the aspect of its own properly defined range of clientele.

From the aspect of annuity models the following possibilities exist so far:

Competing or non competing providers	Central provider	Competing providers			
Provides the full annuity or only the first part of it (what is phased withdrawal)	Full annuity	Full annuity		Only the first part	
Managing selection problems of the providers	Non existent	By premium equalization mechanism	In mandatory maximum	The deferred annuity is transferred	The deferred annuity is transferred to



			pool	to a central provider	a well capitalized market player
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4.2.1.2. Managing the selection as resulting from choosing the annuity type and the rate of indexing

The selection as the consequence of choice of annuity type can not be managed, in other words if there is such a possibility it must be eliminated (if there already exists such option, and if it does not exist it shall not be offered at all), and instead a precise definition of the type of annuities on offer is required. In practice this is a single life annuity for single individuals, the two lives annuity for married couples, naturally both of the annuities are properly defined. But the choice made by the legislator between these options has an equal impact on every annuity model – we may say that there is a free option (only the choice must not be allowed!) – so I do not indicate it on the figure, since this does not separate the possible models.

The same, selection possibilities as a result of choosing the rate of indexing can not be managed, consequently that must not be offered either. This means it provides the fixed indexing rate at an indexation strategy with investments into bonds indexed to inflation, and in the case of a return-refund indexation strategy it means fixing the technical interest rate for the entire market. (Just a reminder: the free choice would be formal here, because due to the competition the providers would be forced to apply the lowest premium, so the possible lowest rate of indexing (the highest technical interest rate). At the central provider the choice among different kinds of indexing would result in certain loss for the provider.

4.2.1.3. Managing the selection resulting from the free choice of starting the annuity

This may only be allowed if this is a front end guarantee time annuity, or generally if the annuity is made up of two parts: one part is up to a determined age without life risk (annuity certain, or phased withdrawal), and the other part is a deferred life annuity. On this basis the key is that the built-up is either simple annuity (immediately starting annuity of one piece), or phased withdrawal + deferred life annuity; this is one of the ways to manage the selection resulting from choosing the provider. In this latter case the start of the phased withdrawal can arbitrarily be



delayed, the payment can be suspended, and naturally this part can be left as a legacy too.

In both cases it is valid that:

1. concluding a contract for simple (immediately starting) annuity and deferred life annuity may not happen at a time selected by the client, that must be tied to a general rule (expediently: the date of retirement)
2. the suspension of the already started annuity may be allowed, that does not cause auto selection

Therefore below I suppose that the first criterion is met, there is a possibility for suspension, and I do not separately indicate this. Though the possible models are divided into simple or deferred annuities (in this latter case it operates as a phased withdrawal until the maturity of the deferral – this is separately regulated!). Above I have already written about such division of the annuity, I gave details about the management of this problem too, however the spitting of the annuity is an issue in the other models too, due to this problem.

The following possibilities exist so far from the aspect of annuity models:

Competing or non competing providers	Central provider		Competing providers					
Provides the full annuity or only the first part of it (what is phased withdrawal)	Full annuity		Full annuity			Only the first part of it		
Managing selection problems of the providers	Non existent		By premium equalization mechanism	In mandatory pool		Transfers the deferred annuity to a central provider	Transfers the deferred annuity to a well capitalized market player	
The life annuity is composed of one or two parts	One	two	One	two	One	two	two	Two

4.2.2. Who bears the mortality (including the loss due to the lengthening of the life time) loss?

Either the client or the provider may bear the mortality loss, or they may share it among themselves (so the loss that still remains despite of the pool). However the provider may only bear the loss, if the its ownership does not coincides with the



clientele, since if it does, the loss is still born by the clients – at most it is redistributed among different groups of clients in incorrect manner. So the bearer of the mortality loss depends on whether the owners of the providers are separate from the clients or not. In the previous case, the for-profit providers were described, who bear the loss directly against the mandatory solvency capital. In the second case neither the for-profit orientation nor the required solvency capital have sense (what is not even a required from the Funds by the Hungarian fund-regulation).

So the possibilities are:

1. the provider is a for profit organization and
 - a. entirely bears the (final) mortality loss
 - b. the mortality loss is shared among the provider and the client
2. the provider is a non for profit organization (⁴⁶=owned by the clients, e.g. in such a case the “members”, it has no capital, or it is a central provider). In this case the (final) mortality loss is entirely born by the client.

In a consistent regulation the mortality profit naturally belongs to the person who bears the mortality loss as well, e.g. in case 1.a it belongs to the provider, in case 2. it belongs to the client. According to the above said case 1.b is meaningful, if the traditional, return refund technique is applied, and if the index is the combined investment-mortality return index. In this case the provider guarantees that the annuity remains at nominal level, it shares the mortality loss (and exclusively that) with the client to this extent. This applies to the case of the minimum pool, therefore this is justified to be used only in the case of for profit providers (with non-for profit provider it is much more reasonable to use the maximum pool).

In the second case it is useful to apply the buffer technique, in order to moderate the annuity fluctuation. This might be considered in 1.b case too.

It is not reasonable that the central provider be for profit organization, therefore the logical solution in this set-up is that the mortality loss is born by the clients.

Theoretically both the for-profit and the non-for profit competing providers might transfer the deferred annuity to a central, respectively to a well capitalized

⁴⁶ At least if the regulation is consequent.



provider if the annuity is split into two parts, but the non-for profit providers tend to transfer that to the central provider, the for profit providers tend to transfer for the well capitalized market players therefore – for the sake of simplicity – hereinafter this is what I presume.

The following possibilities exist from the point of view of annuity models:

Competing or non competing providers	Central provider		Competing providers							
Provides the full annuity or only the first part of it (what is phased withdrawal)	The full annuity		The full annuity				Only the first part of the annuity			
Managing the selection problems of the provider	Non existent		By premium equalization mechanism		In a mandatory pool		Transfers the deferred annuity to a central provider		Transfers the deferred annuity to a well capitalized market player	
The life annuity is composed of one or two parts	one	two	one	two	one	two	two		Two	
The provider is for or non for profit organization	non	non	for	for	non	for	non	for	non	For

In respect of the central provider (so obviously non-for profit organization) the logical question is whether should it be identical with the Social Security (hereinafter: SoS) organization which pays the benefit from Pillar I. of the pension system. As the SoS is not engaged in investment, as there is the pay-as-you go system in place, therefore it has no advantage if the central provider and the SoS are identical organizations; therefore I do not deal with this possibility, apart from example. Namely it is conceivable that the insured individual is not parallel entitled to the SoS and private pension, but one comes after the other so, that the SoS is due from older age than today. Practically this is one version of the life annuity of two parts, where the first part is entirely the private pension system; the second part is entirely provided by the SoS system. So this can be taken if someone wanted to retire at an earlier age than the retirement age in force today, then he should make savings and this saving would fill up the gap until the starting time of the SoS pension when actually he can retire. The figure may be modified as follows:



The following possibilities exist from the point of view of annuity models:

Competing or non competing providers	Central provider		Competing providers							
Provides the full annuity or only the first part of it (what is phased withdrawal)	The full annuity		The full annuity				Only the first part of the annuity			
Managing the selection problems of the provider	Non existent		By premium equalization mechanism		In a mandatory pool		Transfers the deferred annuity to a central provider		Transfers the deferred annuity to a well capitalized market player	
The life annuity is composed of one or two parts	one	two	one	two	one	two	two		two	
The provider is for or non for profit organization	non		for		non	for	non	for	non	SoS

We can conclude from case 1.b that the owner of the mortality risk is in relation to the indexation technique as well.

4.2.3. Possible indexing techniques

Two of the possible indexing and investment techniques (in close correlation) may be considered in the case of mandatory annuities:

1. indexing to inflation
2. indexing based upon extra return.

In the case of indexing to inflation the investment may exclusively be made into bonds dependent on indexing to inflation, whose maturity structure is identical with the anticipated maturity structure of the annuity portfolio. In these cases the regulation provides precisely the indexing to the inflation (probably inflation + fixed 5), and does not permit that different providers change this rate of indexing. In these cases it is not necessary to fix the technical interest rate, as that fundamentally depends on the fixed above the inflation return of the policies and on the cost level of the provider. In the case of indexing to inflation the provider can not abuse that the client is linked to him, therefore there is no possibility to change the provider in the annuity phase. The indexing to inflation does not properly match delegating mortality to the clients, as this may basically be done via changing the indexation, so the basic



presumption here is that the mortality result is of the provider. If still there is a need for delegating the mortality loss to the clients, it is appropriate to apply the buffer technique.

In the case of indexing based on extra return the rate of the index depends on the extra investment return achieved by the annuity provider. It is not reasonable to set the required minimal return, because that would impose a guarantee on the provider what is very expensive and all in all is probably not worth paying for the client. Apart from this may only be a requirement set for for-profit providers possessing actual capital, so I suppose that at this indexing method there is no set minimal rate for the indexation. In this case the technical interest rate must be fixed. If the regulator finds important that there shall be a minimal chance for indexing below the inflation, and then this fixed rate is small, let us say 0%. In this case the starting annuity provided for identical capital will be smaller than in the case of higher technical interest rate, though the annuity increases more quickly, what is more advantageous for the insured person expectedly with longer life than for someone with short lifetime. As the client is exposed to the annual performance of the provider, it is important that the providers are competing for the already acquired clients too, so in this case it is reasonable to assure them the possibility of changing the provider during the maturity. We might think that by setting a higher technical interest rate one may at least partly protect itself against the lower return, but from the point of view of the provider it is not different from the requirement of a high indexation, both the required guarantees, and one can argue against them with the same arguments as against the required high indexation rate. Changing the provider during the maturity as a possibility is coupled with mandating the technical interest rate⁴⁷ and the mortality tables used for provisioning (ie. It is already the second argument in favor of fixing the technical interest rate). The technique gets along with making the mortality profit part

⁴⁷ Certain experts think (e.g. Ferenc Csordás) that the changing of the provider during the maturity does not require the standardization of the product, since in technical sense that is a revision. Though technically this is true, I still do not deem it an adequate solution, because it is too complicated and not transparent for the clients, and practically makes changing the provider impossible. The lack of standardization imposes danger for the client too, namely that the size of the provision is uncertain, e.g. if one of the providers deducts a big cost from the premium and does not put it into the provision, it does not transfer it either, then the new provider regards the transferred provision as premium and



of the indexation. In the case of non-for profit provider the entire mortality loss must be delegated to the insured individuals, so the annuity may even decrease nominally, in the case of for-profit providers it is reasonable to set restrictions, e.g. to order that the annuity may not decrease nominally, so the mortality loss is shared in this case.

From among the two indexing technique one sets the emphasis on the (inflation) safety of the pensions, the price of which is that the pensioner does not receive the results of the economic growth, the other one is (the extra return refund) that the result of economic growth is shared with the pensioners though a part of the safety is sacrificed. The consequence of this is that when the economy is weak, the interest of the policies will be the best, so the pensioners are best off in this situation, at the time of an economic boom the pensioners are worst off, since the state does not have to pay much for the loans. So indexing to inflation protects pensioners against the economic fluctuations at the price that they do not enjoy the impact of the boom either.

In the case of competing services the mandatory pool does not go well along with indexing to the inflation, since among this condition there is no room left for competition, so in this case I suppose indexing based on extra return.

The following possibilities exist from the point of view of annuity models:

Competing or non competing providers	Central provider		Competing providers											
Provides the full annuity or only the first part of it (what is a regular withdrawal)	The entire annuity		The entire annuity					Only the first part						
Managing the providers selection problems	Does not exist		Premium equalizing mechanism		In mandatory pool				Transfers the deferred annuity to a central provider				Transfers the deferred annuity to well capitalized market player	
The annuity consists of one or two parts	One	Two	One	Two	One		Two		Two			Two		
For profit or non-for profit provider	non		for		non	for	non	for	non		SoS		For	
Indexing	To inflation (inf.)		Extra return (er.)		Extra return				inf.	er.	inf.	er.	inf.	er.

4.3. POSSIBLE ANNUITY MODELS

Based on the above table we may distinguish 14 annuity models, however it is reasonable to merge some models (and the different arms may be managed as

again deducts the cost from that. So by changing the provider the client may quickly come to the end of his money and annuity.



internal alternates). I think that wherever the annuity may consist of one or two parts, respectively where both indexing methods may be considered these should be looked at as internal alternates, so I put these two lines at the end of the above table:

Competing or non competing providers	Central provider	Competing providers					
Provides the full annuity or only the first part of it (phased withdrawal)	The full annuity	The full annuity			Only the first part of it		
Managing the providers selection problems (pool, or transfers the deferred annuity)	Does not exist	minimal pool	Maximum pool		Transfers to a central provider		Transfers to a well capitalized market player
The provider is a for profit or a non-for profit organization (the final bearer of the mortality risk)	non	for	non	for	non	SoS	For
The life annuity consists of one or two parts	Both are possible	Both are possible	Both are possible		two		Two
Indexing	Inflation	Extra return	Extra return		Both are possible		

So the choice have decreased to seven models, of which each has one (and based on the above only one) internal alternate either from the point of view of splitting the annuity or from the point of view of indexing. For the sake of further simplification I summarize the two models of maximum pool, so the cases when non-for profit, respectively for profit provider provides the service. These models may be called by the following names (later I give more explanation).

1. the model of the central provider
2. the model of insurer's annuity
3. the model of the central pool (with funds or fund managers)
4. the model of the fund's annuity
5. alternate SoS model
6. OECD model

From among the possibilities the Chilean model – proposed by the World Bank - is missing (at least in its pure form), which combines the indexing to inflation with the competition of the for profit providers. This may be done in Chile, because there is not prohibition on distinguishing among the insured people, so the selection problem to be managed from the start does not exist there. The selection problem can be managed either by the (minimum or maximum) pool, and consequently by indexing the fund of



extra return, or by splitting the annuity. Naturally the OECD model can also be regarded as a specific version of the Chilean model, since they apply indexing to inflation.

The “OECD” model is slightly misleading because the OECD mostly suggest to split the annuity so, that up until a high age (they mention 80-85 years) there would be only regular withdrawal, and then the deferred annuity. So all of such models, where there is a splitting could be called “OECD” model. I did not do it because in the other models – at least as I feel – there is such a model item, what is more “striking” than splitting the annuity, so I stick to this characteristic in the naming. This is partly true for model No. 3 and 4, as both (respectively the non-for profit version of No. 3) could be called “fund’s” annuity model, be in the No.3 I find the central pool a much more important characteristic than the fund’s participation in the annuity service.

Below I investigate these models in detail one by one. I will examine certain general problems that are true in respect of the different models. These will be examined at the description of models in order of occurrence.

4.3.1. The model of central provider

4.3.1.1. The essence and internal context of the model

Based on the above the most important items and their internal context is the following:

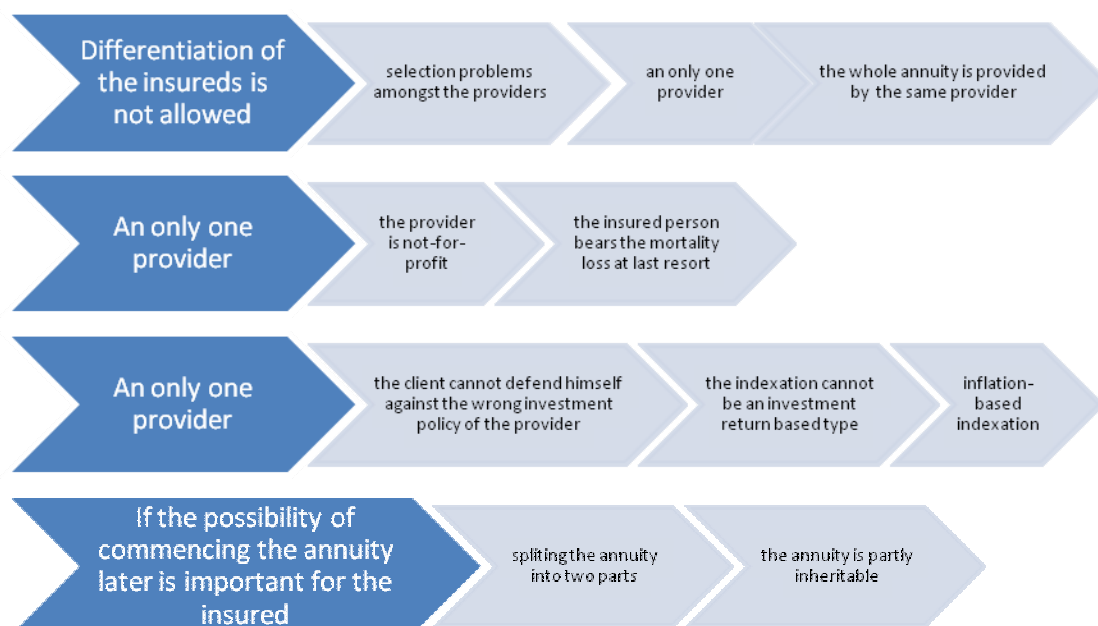
Competing or non competing providers?	There is a single central provider, so selection problems resulting from choosing the provider do not occur.
Does it provide the full annuity or only the first part of it (regularly money withdrawal)?	Naturally, the single provider is “forced” to provide the full annuity.
Is the annuity composed of one or two parts?	The single provider does not mean that the annuity can not be split into two parts – not due to selection considerations, but in order to allow insured individuals have legacy and deferral possibilities. Though this is an option here, both are possible but one must choose between them, so either this or the other



	option is applied on every insured person. So when they retire, they uniformly receive either immediately starting annuity (though that may be suspended), or deferred annuity plus phased withdrawal possibility – among flexibly manageable wide limits.
The provider's relationship to the profit i.e. the final bearer of the mortality loss.	<p>The central provider should not be privately owned, so the mortality loss (respectively the entire mortality performance) must be finally born by the client himself. As in the case of the central provider the indexing based on extra return should be excluded, therefore the mortality performance can not be spread by regular indexing among the insured persons. The possibilities merit of consideration:</p> <ul style="list-style-type: none">• Loss and profit is accumulated against the capital of the central provider for a while, and therefore the service is adjusted from time to time (less frequently than a year)• The buffer technique is applied, by this is achieved that the above solution shall be rarely or shall not be applied at all• Option premium is paid to the state to take over this risk
The possible indexing	<p>From among the two possible indexing techniques that one should be excluded, where the index is not pre-set, but it is a post factum value depending on the investment performance, because with a central, monopolistic provider the client loses every possibility to stimulate the best possible performance. Therefore the only indexing should be the one based on investment into bonds with yield indexed to inflation</p>

The above options can be presented in the following figure:

Figure 3: The internal context of items of the model of central provider



4.3.1.2. The possible owners of the central provider

If there is a single central provider, that can not be a for-profit, privately owned provider. It would be difficult to justify if a monopoly was transferred to a market player, especially when – as it is the case in Hungary – there still aren't market players. In this situation the monopoly would be given to such a market player who has not yet proved that it deserves this privileged position. An other argument against the state's commissioning a market player (let us say by preliminary tendering) to develop a monopoly, that in such a situation the state gets into a blackmailed situation. Namely it can not have a strong stand against the provider if it would not give a service of acceptable quality, or would provide the service at high cost, because the provider may threaten with withdrawal from the activity, while the state being responsible for the service but having no means for that would get into a difficult situation. So it is predictable that the state would have difficulties with ordering a monopolistic market player, so this situation should be avoided. Due to these arguments I would suppose at this model, that the central provider means a non for profit organization.

As for the ownership there still is an alternate solution. The owner may be:

1. the clients similarly to the actual Hungarian funds
2. the state



It the state is the owner, it is logical to ask, whether the central provider should be established by extending the activities of an existing state owned organization, or should it be a new, independent provider? In the first case the logical candidate is the actual SoS, the pay-as-you-go system. Both solutions are reasonable, but the SoS can only be a solution if the central provider is not supposed to make investment activities (see “alternate SoS model”). If it must conduct investment activities too, there is no advantage in building up on the existing organization, it is better to have a separate organization.

Though if a separate organization is established, it is to be considered, that the stakeholders monitor, so there should be a kind of fund – in Hungarian sense – with assuring theoretical possibilities for the stakeholders to directly have a say in the management the affairs. An argument against this solution is that according to the experience the activity of the member of the actual Hungarian funds is a minimum, so the possibility to have a say may even be illusory, respectively the theoretical possibility evolves for an active minority to acquire the control over the organization.

Managing the mortality loss in different ownership structures is basically similar, since in each case the final bearer is the insured individual. If the state had the ownership it may theoretically raised that the final bearer of the loss shall be the state, i.e. the taxpayers, but this is not an equitable solution neither if the state gains nor if it loses on it. In principle there is still a possibility to spray the loss among different generations of the annuitants, but it is neither an equitable solution, therefore it can not be proposed.

4.3.1.3. Theoretical problems of nationalizing the private pension system

The statelty hold provider (but even the one owned by the members, but centralized one) means the nationalization of the private pension mode. This would be a step backwards in such a situation – what the current Hungarian is – where the accumulation phase is based on the competition of non state owned providers. Therefore it should be examined when and why the idea of a central provider may be raised. In tow major cases:

1. if it is more *reasonable* for the state to organize the service, than in the from of competing market providers



2. if the state itself is *forced* to organize the service in the lack of market players

A further breakdown of the above cases:

It is more reasonable for the state to organize the annuity service if

- a) it was not reasonable to entrust the non state owned, competing providers with the service even in the accumulation phase
- b) the phase of accumulation from this aspect is different from the annuity payment phase

At the time when the institutions accumulating private pension assets were generally established in Central- Eastern-Europe the major goal was to mitigate the unavoidably threatening financial pressure of demographic nature on the pay-as-you-go systems, the intention was to recapitalize the pension system. The recapitalization brought forward the latent deficit of the pay-as-you-go pension system and spread it for a longer period of time, by this assuring some chance for the state to better manage it. The state uses this chance at the time when the capital in the recapitalized system does not fundamentally mean its own debt bonds. If this is the situation it means that the state did not take this chance in reality, or if we are more benign it did not take this chance. This also meant that nothing happened in favor of the main goal: the implicit sovereign debt was replaced by explicit one, what is a worse situation than the starting point was⁴⁸. If this happens, as it happened in Hungary too, all the criticism against the private pension system is justified, namely that was established unnecessarily. However in this case the problem occurs not with the competition itself (though it may only be restricted), but with Pillar II itself: If it was created so it was in vain to establish it at all. The system would not have been either better if a single, state owned provider was established. Though if we say that on long term the private pension institutions do not invest into own government bonds, then it is justified to request that the clients who bear the investment risk get the possibility of choice in

⁴⁸ Only out of curiosity: even if the state mandated that the private pension accumulating institutions invest into own government bonds, it would be a complete own goal, respectively!!! Naturally we can see such international practice too, what is not an own goal: some African countries mandated that pension institution invest the capital of the pensioners (e.g. civil servants) into government bonds of return below the inflation rate. This means robbing the pensioners from the aspect of the state, which means it has an objective, however I exclude such motivations from my study.



exchange for the risk taking. The possibility of choice can not only be offered by competing providers but also by a single provider too, but there is no cost pressure in this case. Consequently the competition should be left for competing providers not on portfolios within one provider. The state is not the adequate (majority owner) of the competing providers, so I think that it was right to leave the service on non state owned, competing providers in the accumulation phase.

Naturally this is a rough statement; because it does not take into account that

- a. what precisely the competing providers compete in?
- b. is it really worth to make each component of the activity compete?

In the accumulation phase the competition (provided that the provider does not give guarantee on the return, as it happens in Hungary in the Private Pension Funds) has two areas:

1. the net return achieved by the provider
2. costs accounted for by the provider

The Private Pension Funds generally name a vague third factor “the quality of the service”, but it is not really operational since the service is unambiguous, so the basic functions (registration, notification of the members, etc.) can only be done in two ways: well or badly, but it is a basic principle that the provider carries out this functions.

The second field of the competition is the cost accounted for by the provider what includes the administration, organizing and operating the membership registration. This can be characterized by a strong scale of return because the fixed costs make up a large amount. The bigger the provider the incurred costs are the lower. Therefore it may occur that though a provider has a competition advantage in respect of investment, it is not necessary the case in respect of the administration. Consequently it is not certain that it was justified to set up the system so that its every factor is of competing nature, it is possible, that a centralized membership registration and administration and competing investment providers would have been more reasonable.

According to the provisions set by the law the possible fields of competition in the annuity payment phase are:



1. net return achieved by the provider
2. costs levied by the provider
3. the tailored made nature of the annuity provided
4. longevity risk management
5. long term solvency of the provider

There are more potential fields for competition in the annuity payment phase than in the accumulation phase. However there are a number of arguments for the state to regulate this competition in these fields. In the annuity payment phase it is much more important to have a calculable return adjusted to inflation than to have a high return achieved by the provider. In a mandatory system the annuities may not be indefinitely variable, so most probably the regulation should restrict the range of possible varieties, i.e. the competition in tailor making (as I have indicated above this is much more about avoiding the competition). The longevity risk is often delegated the provider, what is compensated in the costs, the competition in this field is transformed to cost competition. The long term solvency of the provider is much more important in the annuity payment phase than in the accumulation phase as; in this latter case there is no commitment while in the payment phase there is what the provider may eventually not fulfill. What remains is the competition in costs and in the long term solvency of the provider. The scale of return discussed in the context of accumulation phase is also applicable in respect of the fee for administration i.e. the input cost, therefore its central management should be considered.

The long term solvency is influenced in a positive way if:

1. there is such a solvent owner or sponsor behind the organization like the state
2. if the risks are in a single pool, so the random risk fluctuations cancel each other.

In the case of annuities a central provider is a logical alternate, what does not contradict the goals of the private pension system. However this is a factor in the context of politics, what is an argument against. If the population has exaggerated expectations to the national budget what is regarded as the tool of welfare, at the expense of the economy. From this aspect the national budget should be alleviated



from a part of the welfare expenditures (so the pension system). In the model of competing non state owned providers this alleviation takes place while it does not in the model of the central provider; the central provider – whatever regulation is designed – remains in wider sense and in reality part of the national budget. In addition it much more allows than the system of competing providers that the Private Pension System similarly to the SoS system becomes victim of irresponsible political promises.

Naturally the model of central provider does not only emerge if the legislator finds it more reasonable, but also if there aren't sufficient enterprises that are ready to rationally organize the private pension annuity, so in the case of a mandatory private pension system the state may be force to set up a central provider. Such a force is better be avoided, if the state establishes a central provider it should it because it is more rational not because there is not other option. . So the state should avoid that it becomes its own fault that there is no voluntary annuity provider. The state may cause this even if it raises inconsistent and therefore non-compliable requirements against the providers. An example of the Hungarian annuity legislation in force (in 2009, otherwise inconsistent): the provider must increase the annuity at least by the Swiss indexation in each year. As there is no capital market facility whose return would be in good correlation with this index (within this the wage index), therefore the providers can meet this requirement if they make extremely high provisioning, so they would set a very low starting annuity. As the legislator requires not only the adequately indexed annuity but also the one with adequate level, in this situation the possible answer of the legislator may be to choose cost maximizing, what together with the inadequate indexation rule makes almost impossible that prudent providers enter or remain in the market.

4.3.1.4. The problems of mandatory investment into bonds indexed to inflation

In the model of central provider the premium reserve is invested into bonds the return whereof is exclusively dependent on the inflation. Naturally this investment and indexing technique may not only be used in this model, but wherever we use it the problem of nationalization emerges, somewhat differently than in the case of institutional centralization via the establishment of a central provider. The investment



into bonds indexed to inflation specified for Hungary (and probably for many other countries too) means mandatory investment into government bonds, i.e. the implied nationalization of annuity provisioning. Therefore the question is raised whether instead of implied nationalization should not the annuity provisioning be openly nationalized, so to set up a central annuity provider?

The consistent models I found, suggest that the answer is yes, because I only deem the indexation to inflation (beyond indexing to the excess return refund as an alternate) feasible in such models, where the annuity is split, and the deferred annuity, meaning the peak risk is either transferred to a central provider or to a strong market player. In fact at this latter solution it is doubtful whether the indexing technique is logical and consistently applicable “as well”.

Nevertheless it is feasible that on long term investing into bonds managing the inflation would not mean investing into government bonds, or at least not into domestic government bonds, as

- hopefully we join the euro zone in foreseeable future, and the foreign sovereign securities and company bonds indexed to inflation will be available for us
- it is feasible that following the example set by the state the market starts to issue bonds of adequate quantity and quality and content indexed to the inflation, as it happened in the case of Chile.

Naturally the euro zone inflation indexed bonds would not necessarily solve the problem, since the rate of the inflation may be different in different countries within the euro zone. Naturally there is a trend for equalizing of the inflation rates in a common market, if the level of development of the different countries is not too far away from each other. Until Hungary catches up with the average of the euro zone – and this may take decades – the prices are characteristically lower here, than in the more developed countries what means that the prices catch up too, so the rate of inflation will be higher here than in the more developed countries of the euro zone. The indexing to the inflation in the pension system should be linked to the domestic inflation and by this the using other countries’ inflation managing bonds may – not necessarily – be a problem.



It should be also noted that in the case of investing into inflation managing bonds the requirement is not only that the yield of these bonds be dependent on the inflation but also that

- a. these must be denominated in the same currency as the liability is, so in the currency the annuity is paid
- b. the maturity structure of the bonds must adjust to the maturity structure of the annuities (asset-liability matching) so they may not be any kind of maturity, or short term bonds only.

This latter requirement must be respected in order that at the valuation of the annuity provisions the market fluctuations of the exchange rate of the bonds can completely be disregarded, and only bonds shall be in the investment portfolio that are kept until the maturity.

4.3.1.5. The model of central provider in the technical literature

The notion of the central provider emerged only recently in the English language literature, often only as an idea to be refused. For James (ed.) [1994] it was still clear, that a decentralized savings system need private annuity providers. Examining the institution structure of the annuity market Blake [1999] already mentions the state monopoly as a possibility. In his view this may solve a number problems mentioned, the advantages: scaled return, bearing the improvement of the mortality, indexing and the interest cycle could be smoothed. He mentions as the major disadvantage, that there are not many international examples for efficiently managed state owned institutions. This may over compensate the advantage of the scaled return.

For the authors of the OECD, who collect the ideas from many countries at the doorstep of designing the annuity system, the idea of a central provider emerged. In 2007 Fiona Stewart only raises among the options that the state may itself sell annuities. A year later Pablo Antolin also looks for an adequate provider because he thinks that those who are the best prepared for that are the insurance companies but they are not really interested in the possibility therefore the idea of a single national provider is raised, though in his view this may have a superseding effect in respect of the private providers (Antolin [2008a], respectively OECD [2008a]). He adds in his next study (Antolin [2008b], respectively OECD [2008d]) the disadvantage is that the



taxpayers are made the final guarantor, respectively that the state takes back the investment and the longevity risk, what was earlier left out of its portfolio. In addition there is a political risk too, what the state provider can withstand more difficultly than the private providers.

In the Hungarian market the idea of a single central provider was raised by János Stahl in 2000 (Stahl [2000]), at that time only as a possibility to solve problems caused by the unisex table, though in 2005 it was raised as a concept (Stahl [2005]). In 2000 he still found necessary that the idea of a central provider and the NDC based reform of the Pillar I are kept separate from each other, saying that (page 221) “The fact that the previously envisioned system is modified so that there is a single provider, what certainly is a kind of state institution, is naturally not identical with the idea that everyone’s pension contribution is kept on denominated accounts, and the interest is based on the yield of the some government bond.” In other words: this is a caution, that the single provider does not mean that the provision is mandatorily invested into government bonds, nonetheless it is not done explicitly so.

In 2005 he already considers who that central provider might be, and thinks (Stahl [2005]) that such might be established jointly by the funds, or that might be created on the basis of the Guarantee Fund which had lost its function in his view. In the dispute starting with Stahl’s article [2005] György Németh (Németh [2006]) finds Stahl’s proposal justified for a central annuity provider but in his view it needs refinement. In the course of “refinement” Németh proposes a completely different concept, what later will be discussed under the name “model of the central pool”.

The notion of the central provider was for long a “theoretically possible, but practically out of question” possibility until the summer of 2009. Then at a work team session established by the Ministry of Finance for the annuity regulation the idea was unexpectedly supported by Gyorgy Holtzer. Thanks to his influence the bill that included this proposal for the annuity model (though not the way he proposed it) along with an other one got on the table of the Parliament in 2009. Namely he suggested a total renationalization as a solution (in fact it is the version in respect of which Stahl warned that the central provider is not necessarily that!), where the one off premium of the annuity flows into the budget and the annuity – basically like the



pension from Pillar I, is paid from there. According to his vision there is no investment, printing of government securities, separate staff, in essence not even a central provider exists, only there is a central service. The solution is no doubt a cost saving solution, but would reestablish the political risk in the system what was to be eliminated by the establishment of Pillar II, and would circumvent the possibility that the investment would sometime be made into non (Hungarian) government securities, so the Hungarian state would not even be involved in the Pillar II of the pension system via the sovereign debt. (This is temporarily necessary in the model of the central provider.)

4.3.2. The OECD model

4.3.2.1. The essence and internal alternates of the model

The most important items and the internal context of the model based on the above:

Competing or non competing providers	Clients may choose – with certain restrictions - from among several competing providers.
Provides the full annuity or only a part of it (phased withdrawal)	The competing providers only pay the first part of the annuity, the clients may relatively easily structure the phased withdrawal (so it is not, or not necessarily annuity). Among competing providers this partly manages the occurring selection problems resulting from the prohibition of discrimination among clients. .
Is the annuity composed of one or of two parts?	As a consequence of the previously said, the annuity is composed of two parts, where the first unit is the phased withdrawal, what permits that the insured individuals delay the starting of the annuity, and the bigger part can be potentially for legacy (if they do not live it up in the meantime, as it is often referred to “the client has disposal of the money”). At the same time the insured individual must immediately buy the deferred annuity part at the time of retirement.
Managing the selection problems of the provider	The provider is totally void from the remaining selection problems - that are born by the deferred annuity portion - which

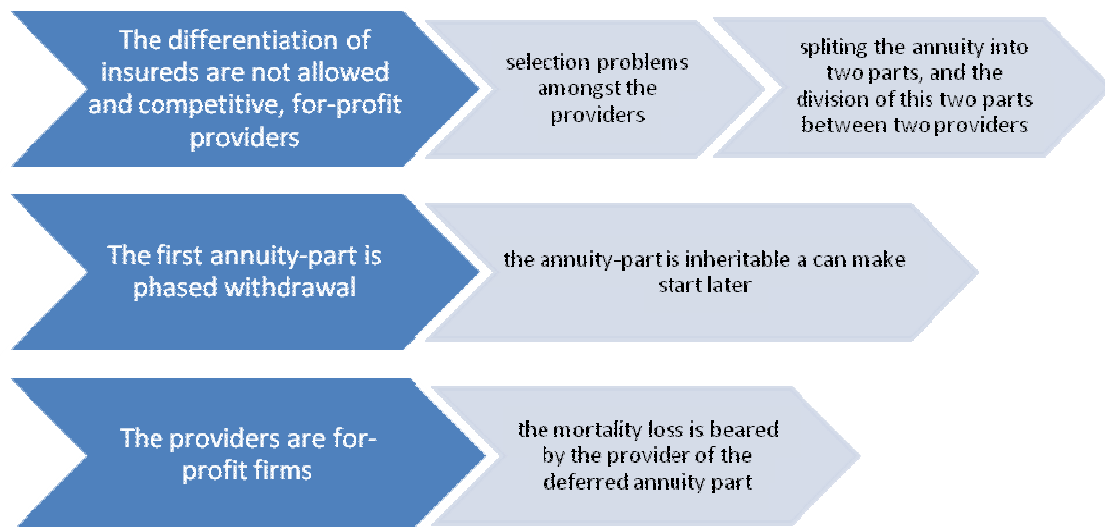


	are smaller from the start than if the provider gave immediately starting life annuity in itself, by passing them on to a well capitalized market player.
The provider is for profit or non for profit organization, it is the final bearer of the mortality loss.	On the grounds of the above momentum, (by passing the deferred annuity to a central provider instead of a well capitalized market player) the provider might even be a non-for profit entity, but this is such a significant characteristics that paves the way for an other annuity model. So in this model the provider is a for-profit company (e.g. a fund transformed to a stock company – among the possibilities given by the Hungarian environment). Accordingly the mortality loss – what exclusively emerges at the deferred annuity portion – is fundamentally born by a well capitalized provider against its solvency capital, which is charged as a kind of option fee in the beginning in the premium.
Indexing	Theoretically it can be solved by both indexing techniques and there are arguments for and against both solutions. An argument for the indexing to the inflation is that the two annuity portions can relatively seamlessly adjusted to each other, and at the version of the deferred annuity portion changing the provider during the maturity is difficult, however without this in the case of the other indexing technique the already „committed” clients are also exposed to the provider in monopoly situation. An argument against it, organizing the service is not really justified by the competing providers in fact it is only raised if the competing providers are taken as given.

The above context can be presented on the figure below:



Figure 4: The context of the OECD model items



The essence of the model is that the client retains his very wide disposal of his accumulated money including the possibility to inherit the yet non used portion (in the event of annuity certain, respectively if the choice is the phased withdrawal), while this autonomy does not endanger the eventual subsistence at a very old age, because from a portion of the money it is mandatory for him to buy a certain “disaster situation” in the form of deferred annuity. The division of the money into temporary and deferred annuity depends on the possible age of the deferred annuity. Below I present a few calculations for the approximate proportions.

4.3.2.2. The splitting of the immediately starting, life long annuity in general

In the OECD models, but not only there, the important element of the model is that each immediately starting life long annuity may be divided into two parts, a so called „temporary „ annuity lasting for a determined period of time, and such a deferred annuity, where the duration of the deferral is identical with the duration of the temporary annuity. Together the two annuities precisely make the immediately starting, life long annuity.

According to the above said the formal splitting is reasonable if:

1. the management of two annuity parts are different from each other, e.g. they are performed by different organizations, as it is in the OECD model, or if



2. if the intention is to keep a portion of the annuity at the disposal of the clients, so instead of the immediately starting simple annuity the general one is the guarantee time annuity. The annuity portion during the guarantee time is a separate and relatively “easily” managed one.

For the provider the biggest manageable risk of the annuity is the longevity risk. If the original annuity is split the original longevity risk is also split but it is not equally spread on the two parts of the annuity. It is easy to recognize that the longevity risk is mainly related to the deferred part of the annuity, as its rate can not be precisely defined, the insured persons may live almost any long compared to what was designed by the provider. Contrary to this the possible rate of loss in temporary annuity portion of the provider can be limited: the worst that may happen is that the insured person does not die during the maturity of the annuity, so the life annuity behaves as annuity certain. To demonstrate the possible rate of loss I indicate the relationship of net premium of the temporary annuity certain and the temporary annuity, I use the male and female select mortality tables of 2000 and 2006 for the calculation:

Temporary annuity premium function of the annuity certain premium								
Age of entry	65							
Order of necrosis	2000				2006			
Technical interest rate	0,0%		2,9%		0,0%		2,9%	
Period/maturity	15	20	15	20	15	20	15	20
Male	71%	61%	73%	65%	74%	64%	76%	68%
Female	83%	74%	85%	78%	86%	78%	87%	81%

It is obvious that the worse the mortality the bigger the potential loss (i.e. the difference in the premium of the two types of annuities) namely it is bigger at men than at women, it is bigger when calculating with the mortality table of 2000 than of 2006 (improving trend compared to 2000). As the annuity is expectedly calculated with a select mortality table significantly more favorable than the projected select mortality table, therefore the possible loss in the temporary part will probably be even smaller.

In addition the longevity risk may be totally eliminated in the temporary annuity part if we calculate with annuity certain from start and not with temporary life annuity. This two annuity parts if regarded together is nothing else than the supposition of a front end guarantee time life annuity insurance. It is of course possible that the annuity amount is somewhat smaller than in the case of the pure life annuity, but the better the mortality this decrease the smaller – as for the rate this table gives hints.



The table below gives information about the relative weight of the two annuity parts, I present the size of the premium of the deferred annuity parts primarily bearing the longevity risk within the premium of the original annuity.

The proportion of the deferred annuity within the premium of the lifecycle annuity								
Age at entry	65							
Order of necrosis	2000				2006			
Technical interest rate	0,0%		2,9%		0,0%		2,9%	
Deferral	15	20	15	20	15	20	15	20
men	18%	7%	13%	4%	20%	8%	15%	5%
women	25%	11%	19%	7%	27%	12%	20%	8%

It can be stated, that the bigger relative premium of the deferred annuity the better the mortality conditions. So in the case of women it is better everywhere, than in the case of men, and in the case of a generally better mortality table (as the one of 2006 compared to 2000) it is bigger, than in the case of a generally worse mortality table. However the bigger the technical interest rate the smaller this proportion ratio is (since the higher technical interest rate makes relative less valuable the elements further in time of the a cash-flow), and naturally the bigger the deferral it is smaller than that (so smaller at 20 years, than at 15 years). However even in the extreme case presumed within the table (presuming women mortality table of 2006, 0% technical interest rate and 15 years of deferral⁴⁹) within the premium the proportion of the premium of the deferred annuity is only 27%. True, it is an underestimated value to the extent, that the actual annuity would be calculated with a –from mortality aspect - better projected mortality table than the 2006 select mortality table when it comes to.

4.3.2.3. The possible temporary annuities

The obvious tool for consuming the private pension saving is the provision of temporary annuity. There are different types of it, therefore the question is raised precisely what sort of annuity shall that be, respectively, shall the consuming be made in the form annuity (what's more, shall the saving be necessarily consuming?). The following options are possible:

- temporary life annuity
- temporary annuity certain



- scheduled money withdrawal
- withdrawal in lump sum

The advantages and disadvantages of the different possibilities:

Possibility	Advantage	Disadvantage
Temporary annuity	<ul style="list-style-type: none"> • since the provision of those who died in the meantime is distributed among the still alive insured individuals, therefore this assures the highest level of annuity from among all the possibilities • it makes a schedule for the insured individuals for the consuming of their money, by this diminishing the risk, that they consume it before the time (presuming undisciplined pensioners with restricted foresight or living under the pressure of needs) 	<p>Not flexible so:</p> <ul style="list-style-type: none"> • it is not inheritable (since the money of the dead is divided among the other insured individuals) • the possible dates of the retirement should be restricted with respect to the danger of self selection • may be provided only by an insurer type of institute
Temporary annuity certain	<ul style="list-style-type: none"> • flexible, so: <ul style="list-style-type: none"> ◦ inheritable without any problem ◦ there is no danger of self selection, so the date of retirement may totally flexibly be chosen • it makes a schedule for the insured individuals for the consumption of the money, by this diminishing the risk that they consume it before the time • may be provided by any kind of financial provider (e.g. the actual Hungarian pension funds) 	<ul style="list-style-type: none"> • since the money of the dead people is not distributed among the still living ones, therefore it assures a somewhat lower level of annuity compared to the temporary life annuity
Scheduled money withdrawal	<ul style="list-style-type: none"> • even more flexible than the annuity certain, permits more possibilities for changing during consuming the money • may be provided by any kind of financial service provider (e.g. the actual Hungarian pension funds) 	<p>There is no default schedule in the course of consuming the money, so there is a danger that the insured person suffers deficit in certain periods of time (assumes more disciplined pensioner)</p>
Withdrawal in lump sum	<ul style="list-style-type: none"> • the most flexible solution since anything can be made with the saving e.g. annuity can be bought in the free market (either annuity distinguished by gender, or life long) or one can start up an enterprise, etc. (so it supposes 	<ul style="list-style-type: none"> • should the pensioner not be disciplined, he may easily run out of money before the annuity starts from Pillar I. • there may be a pressure on the pensioner from the environment for

⁴⁹ So the insured individual would start to get annuity from one of the providers at the age of 65 what would last for 15 years, that is until the age of 80 – if he lives to the age - , and at the age of 80 an other provider starts paying the deferred annuity lasting until the end of his life this time.



Possibility	Advantage	Disadvantage
	disciplined pensioner with adequate foresight, not living under the intense pressure of needs) <ul style="list-style-type: none">• can be provided by any kind of financial service provider (e.g. The actual Hungarian pension funds)	not using the money received as a lump sum to cover his own needs, but e.g. his children's needs

The regulator may decide which of the above possibilities to impose from the top to the bottom. The rule says that the possibilities before the imposed ones must also be allowed though the ones after must not. So if the regulation provides the temporary life annuity, all the other possibilities must be excluded. Though if the regulation allows for the withdrawal of the money in lump sum there is no reason for the prohibition of the other possibilities.

In the OECD model and in the other models splitting the annuity into two parts the possibility for the scheduled money withdrawal – with certain restrictions – is the default, what permits the temporary certain and life annuity too – should the insured individual choose that but it excludes the withdrawal in lump sum. This latter one may be possible above a certain amount – if the legislator wants to homogenize the annuity. A possible and reasonable rule for the scheduled money withdrawal is that after the retirement such proportion of the value at the beginning of the year can be consumed in each year, as many years are before the starting of the SoS pension. The advantage of this solution is the flexibility what does not endanger the potential subsistence of the pensioners (either due to lack of discipline, or due to need or pressure of the environment) though leaves the way open for inheritance. According to the experience it is a widely known and needed feature of the private pension system, probably this feature assures the attractiveness. A further advantage is that the providers may remain the actual funds (this is the annuity model of funds!), so there will be default provider on a system level, contrary to the case when life annuity would be mandated by the law. Namely in the case of mandating life annuity the actual funds should either be reformed into providers of insurance type (with actual owners and solvency capital) or the pensioners must be force to change the provider at the time of retirement (so they should leave their previous funds), which means there will be no default provider – as a potential danger there will be no provider at all if nobody volunteers to enter the market.



4.3.2.4. The duration and the service of the temporary annuity

Contrary to the alternate SoS model, where the duration of the temporary annuity is fixed to the proportion between Pillar II to Pillar I., this is totally optional at the OECD model and at every model where the capital within Pillar II is split. However a few considerations should be made when making the option. Among these the most important is the magnitude of loss, what the annuity system is able to absorb as the free option, respectively the price of the possibility to inherit. A life annuity is “cheaper” than an annuity certain (or the scheduled money withdrawal, what is of the same character) in the sense that the provision of the dead insured individuals is spent on the annuity of the still alive ones, and not on the inheritance, so it “remains” in the system. The longer the temporary annuity, the more money leaves the system so the assessed temporary annuity may be the smaller. Here is the presentation of the previous calculation, what gives an idea about the magnitude of this loss.

The premium of the temporary annuity function of the annuity certain = how much less annuity certain come to compared to the life annuity from the identical starting capital								
Age of entry	65							
Order of necrosis	2000				2006			
Technical interest rate	0,0%		2,9%		0,0%		2,9%	
Period/duration	15	20	15	20	15	20	15	20
men	71%	61%	73%	65%	74%	64%	76%	68%
women	83%	74%	85%	78%	86%	78%	87%	81%

It is worthy to make a remark to the table that that it probably over estimates this loss, because the mortality table of the annuitants would probably be better than the select mortality table of 2006, that is the difference between the annuity and annuity certain will be smaller than in the above table. Based on this I think that we may think about a temporary annuity of 15 years duration without any problem, but a longer duration is to be considered too.

A further question is whether the length of the temporary annuity is selectable or not, or should the provider set the length. An other question is if there is a regulation, should the length of the temporary annuity or the age of the start of the deferred annuity be defined (e.g. how old will be the insured person when the deferral comes to an end)? The clear answer for this latter question is that the client with his possibility to choose is in harmony with the age of the start of the deferred annuity, since the length of the temporary annuity – if the regulation allows the most



flexible version, the phased withdrawal, what can be suspended and can be restarted – can not be clearly defined.

So the question on the length of the annuity can be reworded so that the client may choose the starting age of the deferred annuity, or should the regulation fix it for the entire market. In favor of this regulation (e.g. uniformly at the age of 80 or 85) is that this may eliminate the risk of self selection⁵⁰, what would result in the increase of the premium of the deferred annuities.

4.3.2.5. Joining up the two annuity parts

I have already stated that the annuity must be defined at the time of retirement in order to prevent self selection. Apart from this the regulation should have two objectives in this model as well in any model of splitting the annuity:

1. It must assure that the two annuity parts join up so that during the entire pensioner period the pension coming from Pillar II is possibly balanced or at least does not decrease in real value
2. It must assure that in the temporary annuity phase, when the insured person has a wide possibility to choose this does not lead to consuming the major part of his savings in the first part of the phase.

The first objective may be achieved by rules on indexing, and partly by rules on determining the annuity. In the case of the first objective the rules on determining the annuities must assure that when the annuity starts the service from the two annuity parts are still in harmony, and the rules on indexing must assure that this harmony remains on long term and the two parts of the annuity do not “shift apart”.

The obvious rule when determining the annuity is that the annuity units are defined at the time of retirement so that it is determined in respect of the entire capital how much that would be as an immediately starting annuity. After this the so determined monthly annuity unit is used for the calculation of the capital portion to be used for deferred annuity. The question is what kind of annuity that shall be. There

⁵⁰ I note that the trend of self selection is not quite clear. Arguments support that primarily those might choose a high age for the starting of the deferred annuity who expect a long life, since the protection of the deferred annuity will be effective for them from a higher age. However those expecting a short life may think about minimizing the amount spent on deferred annuity because they would not use it anyhow, so they might want to maximize the capital for legacy.



are two possibilities: simple annuity and front end guarantee time annuity, where the length of the guarantee time is identical with the length of the deferral of the deferred annuity. The other question is – if the client otherwise may choose, that what annuity should he buy during the deferral, e.g. during the maturity of the temporary annuity porting. This choice determines that the annuity unit is calculated on the basis of simple annuity, or guarantee time annuity, or should it be calculated independently from this on the basis of one of them. Wording it differently: whether should the immediately starting annuity determined at the time of retirement be always without guarantee time; or this should be dependent on the choice of the insured person for the type of temporary annuity, guarantee time annuity (so smaller service at the start) or immediately starting annuity.

As at any option, the problem of self selection emerges, therefore the insured person should not have the possibility to choose here either. Though if the legislator does not assure the possibility of choice, the must determine on the basis of which immediately starting annuity the annuity unit is determined. In the case of simple annuity the annuity unit determined for high age will be bigger. If the client chooses a scheduled money withdrawal, the monthly annuity unit will be smaller than this, so this solution lays a higher emphasis on the safety of the very old age. In the case of guarantee time annuity it may happen, - if the client chooses temporary life annuity during the deferral – that the annuity unit decreases at the start of the deferred annuity.

If the client chooses such temporary annuity, which was presumed at the calculation of the deferred annuity unit, the magnitude of the two annuity parts are in harmony with each other. So the rules of indexing must prevent the shifting apart of the two annuity parts that are in synchronous with each other at the start. In the case inflationary indexing this is automatically met, at least in the event if in the first annuity phase the insured individual chooses the temporary life annuity or the annuity certain, since by his option he underlines that for him the schedule elaborated by the insurer for the withdrawal of his money is important for him. Nevertheless the indexing to inflation is contrary to the logic of the scheduled money withdrawal as in this case the insured individual supports the option for free choice, what is significantly



impaired by the by the restriction of the investment – characteristic for indexing to inflation. So the legislator must decide what is more important for him: the seamless joining of the two annuity parts or the free option of the insured person. Joining the annuities may be assured at the return refund indexing too if the premium provision of the deferred annuity is not immediately transferred to the well capitalized provider, but the scheduled money withdrawal is invested by the insurance provider so and with such return as it is done with the provision of the first annuity part. In this case an agreement is concluded with the well capitalized provider on the tariff as well on the condition that the provision of the deferred annuity is transferred to him if the insured person dies (for this transfer he shall not give a service), or if the insured person lives until the end of the deferral.

The second objective, so that the insured person does not immediately consume the entire provision can be assured by a few simple rules. In the case of temporary life annuity and annuity certain it is sufficient to assure that these annuities can not be annuities that diminish with time, and may not come to an end before the deferred annuity starts. It is not important to define that these may not come to an end later, probably an insured individual thinking reasonably will not find important to receive two different types of private pension annuities sometime in the middle of his pensioner years. The starting date should not be defined either, it may take place at any time after the retirement. In fact if the insured person has sufficient accumulated capital this start may be even prior to the retirement.

The case of the phased withdrawal is a bit more complicated. The same applies to the start as to the temporary life annuities and the annuity certain. Since the point here is that the insurer may voluntarily suspend and restart it, and the money withdrawal may not necessarily take place monthly, or not necessarily in equal amount (though probably this is the default), therefore more general rules are needed here than above. Disregarding that the capital may significantly fluctuate during the maturity depending on the result of the investment, so if (apart from money withdrawal) we presume unchanged (or evenly increasing) capital, the following possibilities are at the disposal of the legislator, depending on what he would like to achieve:



1. If the objective is that the money that can be withdrawn does not diminish from month to month, the possible rule is that no more money can be withdrawn from the capital during the month than the beginning of the month balance divided by as many month as many are until the start of deferred annuity. If somebody withdraws the same amounts regularly and does never interrupt the withdrawal, this rule practically assures him the annuity certain. If he interrupts the money withdrawal some time, the money saved so may be spread evenly on the remaining month, and can be withdrawn. The disadvantage of this solution is that it makes impossible that someone withdraws a bigger amount at once, for example after having not withdrawn anything for a long time.
2. If the objective is to be able to withdraw the (indexed) level of the starting annuity in each month, then a possible rule is that such an amount may be withdrawn at any moment, which is the difference of the existing capital and of the following product: the starting level of the deferred annuity indexed by the investment return until that time and multiplied by the remaining number of months. This rule permits the one off, bigger money withdrawals too.

If the second rule is accepted – so by suspending the money withdrawal, having adequate “preliminary savings” the insured may be allowed to withdraw bigger amounts as a lump sum from his account from time to time. The question is whether the money withdrawal of bigger amount is possible at the time of retirement too. As this question is a general one, applicable for each pension model, therefore I give the answer in the general part.

An other question is whether the already started temporary annuities can be changed within the tree types. The changing between the annuity certain and the phased withdrawal (back and forth) does not have any theoretical (maximum administrative, cost) obstacle. However as the consequence of possible adverse selection the already started temporary life annuities can not be converted into the two other annuities. The danger of adverse selection is much smaller in the case of the



reverse shift, so theoretically there is not obstacle to shift from the annuity certain or from the phased withdrawal to the temporary life annuity.

4.3.2.6. *The OECD model in the literature*

Much before the OECD publicized this model, it emerged in Hungary as a somewhat spontaneous “popular need” and independently from each other many formalized versions were created in Hungary as well as abroad. I said in 2007 that the wide expectation towards the private pension fund to become inheritable, and the possible smallest part of it should be shared in risk community with others. Many people said that the phased withdrawal, as a solution is self explanatory for them, this is the consequence of the logic of private pension funds, and they could not imagine any other type of annuity. “Experts” adopted it to a feasible system that they joined the phased withdrawal with deferred annuity as a “cover for disaster”. This formally appeared in the Hungarian technical literature by Kolos Ágoston (Ágoston [2008]), and in the international literature it is mostly advocated by the OECD. From the part of the OECD this model is an official recommendation for those countries that only design the annuity system of Pillar II. This was officially put forward in Paris, in November 12, 2008 at the event „OECD Seminar on the payout phase of pensions, annuities and financial markets” and it can be found in the then published Antolin [2008a], respectively in the OECD [2008a] publications. According to these two important risks must be managed: the risk of timing of the conversion into annuity and the longevity. The most important payment options of the accumulated capital are the lump sum, and the scheduled money withdrawal, the life annuities and their combination, among these the balance may be found by the equilibrium of flexibility and protection. Therefore the idea is that the best solution is the combination of the programmed money withdrawal and the deferred annuity (bought at retirement). This latter one would start at about the age of 85, what would use some 15% of the accumulated money.

Though Antolin does not refer to preliminaries, the idea has a short history in the literature, though the Canadian Moshe A. Milevsky, who proposed his idea as a very long term deferred annuity, he did not raise it as a model of payment of the formal DC systems. This may be regarded as a new idea because even Blake said in 1999 (Blake



[1999]) that the market of deferred annuities is so small, especially if the deferral is very big, that this market does not exist in practice. If these can still be bought, the conditions are generally very bad. In addition the market is shrinking as a result of mortality uncertainties.

Milevsky in his article „Real Longevity Insurance with a Deductible: Introduction to Advanced-Life Delayed Annuities (ALDA)” (Milevsky, Moshe [2005]) mentions Stephenson’s, J. B. article of 1978 as a preliminary („The High-Protection Annuity. Journal of Risk and Insurance 45(4): 593-610. o.”) In his article Milevsky proposes a deferred annuity of ultra length, what in addition would be with regular premium, and would be indexed to inflation. In his view the targeted segment of the society is the North American population that does not have traditional DB pension. He proves that during the pension savings period with monthly 1 USD the client may get a monthly service of 20-40 USD worth provided, that the waiting time (deductible).is high enough. On long term this may diminish the psychological obstacles of the voluntary annuity purchase in such an environment, when the DC plans push ahead against the DB plans, and the low level of annuity purchase calls for a new solution. The service would start at a very high age, at the age of 80, 85, or 90⁵¹. He mentions that the novelty of the idea is the regular nature of the premium too, as this product already existed as a product with one off premium, but that was a commercial fiasco.

Milevsky’s idea evoked a range of literature. According to Gong-Webb [2007] the product has 3 major advantages: 1. the households maintain their liquidity until the start, 2.this is preferred by the households, 3. by this the consumption of the capital can be managed by simple thumb rules. However in his view it is still questionable whether this product really diminishes the aversion against the conversion into annuity. Therefore he proposes that in the ALDA-t a 401(k) plans the OECD model is made a default. However he notes that this solution is disadvantageous for people of short life expectancy.

⁵¹ My own idea: the unforeseen change of mortality may be managed by the adjustment – i.e. the increase - of the starting age



4.3.3. Alternate SoS model

4.3.3.1. The essence of the model, its internal alternates

The most important elements of the model based on the above said, and their internal context is the following:

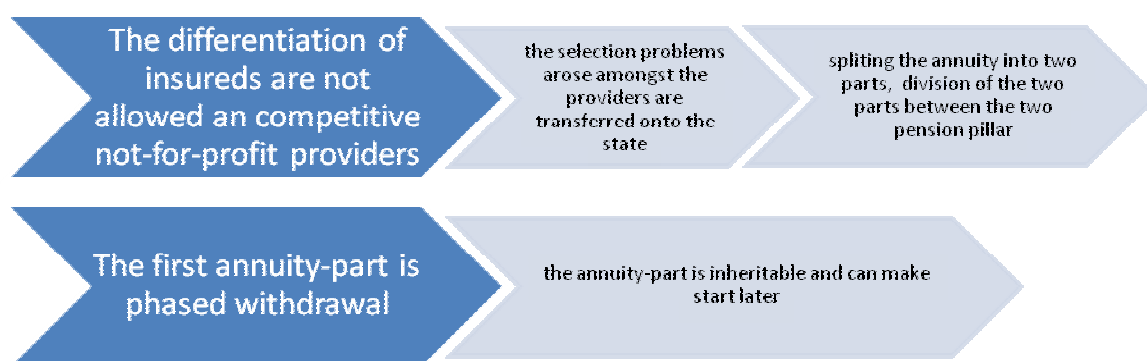
Competing or non competing providers	Several competing providers give service, the clients may choose from among them with certain restrictions.
Provides the full annuity or only the first part of it (phased withdrawal)	The competing providers only provide the first part of the annuity, in the framework of phased withdrawal what is relatively freely designed by the clients (so not, or not necessarily in the framework of life annuity). However this first part is the first part of the annuity of the SoS system, so parallel with it there is no annuity from Pillar I. Though the deferred annuity is fully provided by Pillar I. of the SoS, so there is no more annuity from Pillar II. This structure among the competing providers manages completely the selections problems resulting from the prohibition of differentiation. (respectively it may be maintained if the clients may choose temporary annuity too instead of the scheduled money withdrawal)
Is the life annuity composed of one or two parts?	As a consequence of the above the annuity (what is mandatory old age pension from work) is composed of two parts, where the first unit is the phased withdrawal, what permits, that the start of the annuity be delayed by the insured individuals, and the majority may be potential legacy (if they do not consume it in the meantime) it is generally referred to as ("the client has discretion over his assets.").
Managing the selection problems of the provider	In this structure no selection problem remains among the providers.
The provider is for-profit or non-for profit organization, e.g. the	The issue of the mortality loss is not raised in this model from the aspect that the deferred annuity is of pay-as-you go nature, where this question is generally not examined. The competing



final bearer of the mortality loss.	providers may be for- and non-for profit providers too, however in a situation where (as currently in Hungary) where there already are non-for profit providers, there is no reason at this model to replace them by for profit providers. It must be noted though, that this is not a self explanatory feature of the pay-as-you go systems. The pay-as-you go systems of individual accounts (primarily the NDC) can raise the issue of managing the longevity risk, respectively that it is always credited on the given cohorts. .
Indexing	Theoretically it can be solved by both indexing technique. There are arguments for and against both solutions; It is also possible, that the insured individuals are allowed to choose the investment, by this the indexing technique. However at this model it is almost certain, that the two annuities “shift” apart from each other, since the indexing almost certainly takes place along different principles.

The above context can be depicted by the following figure:

Figure 5: The internal context of the items of the SoS model



The essence of the model is that the national and the private pension (Pillar I and II, or SoS, respectively mandatory annuity) are not parallel in time, but the two annuities are subsequent in time so a pensioner at a time gets annuity only from one source and not from two sources. Naturally both the SoS and the mandatory annuity can be the first one in time, but I do exclude one of these possibilities. Though its is



theoretically possible that after the retirement the SoS annuity starts immediately, lasts until a certain duration, or until a certain age, then (if the pensioner is still alive at the time of the maturity) it is replaced by the private pension annuity, but this would not be a lucky solution. The other possibility is the reverse one, when the pensioner first consumes his private pension savings, then, from a pre-fixed age, or from the end of the term the SoS annuity starts. Naturally this pre-fixed age is higher compared to the actual age of retirement and to the official age of retirement. The above two options are not symmetric because the essence of the splitting of the annuity that the longevity risk is either partially or fully taken over by that provider who provides the deferred annuity. From this aspect the first possibility can be excluded, so that this provider is the provider of the private pension annuity (insurance companies, funds, respectively generally competing providers). Namely the SoS – with the underlying explicit or implicit government guarantee can much better and mainly much cheaper bear the longevity risk than the competing market providers. Taking over the longevity risk for them (finally for their clients) is much more expensive and it is uncertain too which of them is able to bear this risk, so whether some of them does go bankrupt, leaving old people without care behind, for whom the state should finally provide service.

So I suppose in the alternate SoS models that the pension assured by the SoS system (e.g. the mandatory Pillar I. and II. together) is composed of two parts: after the retirement the pensioner first consumes (or not – up to his discretion) the private pension savings, then the SoS pension starts from a later age and lasts until the end of the life.

4.3.3.2. The duration of the private pension, the transition

The question is naturally raised, how long the private pension should last and from when should SoS pension start. Should the turning point between the two be defined at all in age or in the duration of the private pension? Among the two theoretical possibilities the turning point is due

- At a defined age of the insured person – e.g. at the age of 75, or
- At the end of the pre-defined, fixed duration of the private pension annuity.



After a short analysis it can be recognized that from the two options the first one should be selected. The advantage of a turning point at a defined age is that

- Saves the SoS annuity (what is more the whole system, if the temporary annuity is annuity certain) from the adverse selection as the consequence of the option made by the pensioner (i.e. concerning the date of retirement).
- The private pension annuity can be joined with different kinds of SoS systems, contrary to the pre-defined turning point, what can be well joined only with the NDC SoS systems (score system), otherwise problems of equity (and via this problem of incentives) may emerge. The essence of the equity problem is that those who select a later date for retirement should be adequately benefitted by the increasing of the SoS pension, though this can only be done with an accurate survey of the entitlements. (Naturally the SoS system is worthy to be converted to denominated account system, even in the case of a defined turning point, because of the advantages of such systems.)

In the case of both turning points it is true that the system motivates for a later retirement (since the money is accumulated on the private pension accounts, what can be inherited). If the proportion ratio of Pillar II is sufficiently big within the system, the turning point defined in age takes place anyhow at such age, when the majority of the pensioners actually retire, so it is insignificant that the system motivates for the deferral of the SoS pension or not, so there is no advantage in the turning point defined by the duration. Though there is a significant disadvantage in defining the turning point with duration, that it can not be well joined to the SoS pension. By joining I mean that there should not change or should not be a big difference for the different pensioners in the service when there is a changing between the two types of annuities. If we suppose that each annuity part is actuarially correct, than in the case of turning point defined in age the SoS annuity can be relatively well seen by the pensioner. If he decides so that he does not start to consume the private pension savings, then only potential his private pension annuity may be growing – compared to the SoS annuity, but this does not cause a problem, because he is not mandated to consume it. Though if the turning point is defined in duration, we necessarily have to



start from the presumption that the pensioner consumes (starting from sometime, what may be after the official retirement age too, according to the discretion of the pensioner) his private pension savings, so a private pension annuity level can be calculated for him. If the duration is pre-fixed, the annuity level defined for this duration is not necessarily in harmony with the actuarially correctly defined SoS annuity level, as this will be different in each age.

In summary I think that the turning point of private pension annuity and the SoS annuity must be clearly determined in age. The next question is which age it should be. The principle is obvious:

1. The duration of the temporary annuity must be added to the official (minimal) retirement age
2. The duration of the temporary annuity must be determined so that average assets accumulated on the individual accounts roughly assure identical monthly (depending on the regulation life or certain) annuity, what is assured by the deferred SoS annuity on the grounds of the accumulated entitlement.

Together with the determination of the turning point the next logical question is how the transition can take place into such a system. The problem of transition can be broken down into three categories:

1. Only the SoS pension exists but the intention is to split it into temporary private pension and deferred SoS pension
2. The private pension exists already and the deferred SoS pension, and the intention is to increase the proportion ratio of the private pension (i.e. the age limit when the SoS pension starts)
3. The private pension already exists beside the SoS pension, but they are parallel in time with each other.

Generally we can say that the transition is greatly supported is the logic of the SoS pension and the one of the private pension are similar to each other, let's say both are of DC system, so the SoS pension is of NDC system. For my part I only examine this case with the remark, that if the SoS pension is not of NDC system and we desire such a transition, than first the SoS pension should be reformed to NDC system. Then –



disregarding the other problems resulting from the transition, e.g. the concerned people need time to become familiar with the new system, technically the transition can take place overnight in respect of those who still did not retire.

1. The method of transition is that the so far single SoS annuity is split along with the entitlement accumulated so far of the NDC accounts, what are split in the same proportion (so as if the previously single contribution would have gone into two parts from the beginning). The state recognizes as explicit sovereign debt the entitlements so far kept on the NDC accounts, but further on these fall on the private pension, and the state formally issues sovereign debt bonds (in the adequate maturity structure) which serve the basis for the private pension annuity. Naturally this private pension annuity in this structure may only be a (temporary) life annuities the original SoS annuity, so the NDC balances providing the basis were not sufficient for that.
2. The logic of raising the age limit is the same – at least if temporary annuity is assured until before.
3. The transition from the existing parallel annuities may not either cause a bigger problem, in respect of those who are not yet pensioners, providing that the transition for the parallel solution was correct in its time. This for example does not apply to the Hungarian case, since the people who migrated to the private pension system lost all the entitlements they had in the SoS system. The consequence is that the relative proportion of the capital falling on the two types of annuities is different at the time of retirement in respect of different generations: the older the generation the smaller the proportion of the private pension capital (naturally it everything else – e.g. returns – are supposed to be equal). In these cases an increasing age (up to a certain point of time) of turnover shall be determined.

If it is not life annuity but annuity certain which is the target for the temporary annuity, the problem of lack of coverage may be solved so that the transition

- Is postponed by a few years after the splitting of the contribution, and the contribution to the private pension fund is determined so that it covers the costs of legacy too, or



- The possibility for legacy is gradually opened: first only for a short time after the retirement, later this is gradually extended to the entire duration of the temporary annuity.

4.3.3.3. *Joining up the two parts of the annuity*

The two phases of the pension annuity, the parts of the annuity derived from the second than from the first pillar should be joined up, so they should be determined so, that the monthly annuity is the same from both pillars. The two parts of the annuity may be different from each other due to two reasons, so these factors should be taken into consideration at the joining up:

1. The annuity defined at the time of retirement from Pillar I. and II. are a priori different from each other
2. The two parts of the annuity are differently indexed during the period of the annuity of Pillar II, so they “shift apart”.

The joining up appears as a problem only if the service from Pillar II is of actually annuity type, though even in the case of phased withdrawal it may be probable in respect of the majority of the concerned people.

At the time of the retirement, the joining up of the two parts of the annuities can be achieved by the regulation of the retirement age, of which I have already discussed. If Pillar I is of NDC system is a great help, since at the time of retirement (that is when the annuities of Pillar II start) the annuity of both pillars can be calculated according to the same principle on the basis of the accumulated (actual or theoretical) capital. As the accounts of Pillar I and II carry interest according to different logic until the time of retirement, therefore it is not certain that the monthly pension calculated for the two pillars will be identical; the starting pension from pillar II may be smaller or bigger than the later one from pillar I. If the pension from Pillar II is lower than the one from Pillar I is not a problem in itself I think, because the pensioner will experience the increase of his pension, what is generally a positive change. Though if the pension of Pillar II, that is the starting pension is higher we should avoid that the pension decreases at time of transition to Pillar I, what can be simply done if the system allows that at the time of retirement, or later the pensioner (according to his discretion) withdraws coverage of the access annuity as a lump sum.



The “shifting apart” of the two annuity parts can be prevented if the rules on indexing in the two pillars are identical. The following indexation possibilities can be considered:

- Indexation in Pillar I is a matter of decision to a certain extent. The most customary indexing is the price index, respectively indexing to the wages, or the combination of the two. The Hungarian pension system seems to move towards the price indexing; though this is definitely dependent on the politics therefore it is uncertain. The NDC system somehow attracts indexing to the wages, because it allows a fairly simple connection between the increase of income and liabilities, since one of the most important factor of the increase in the income is the wage index. Indexing the NDC also depends on whether the target of the NDC system is self-supporting, so that the ever expenses and the ever income are in harmony with each other on long term, or not.
- Indexation in Pillar II must depend on the investment performance. The rate can be linked to some of the macro indexes (wage index, price index, growth of GDP, stock exchange index, etc.) if the investment itself is fixed, and made into such facilities, that result in return depending on the given macro index:
 1. shares (securities representing a fraction of ownership in a business) in a composition corresponding to a given stock exchange index. Though the stock exchange index is too much changeable to be an adequate basis for the indexation of pensions, not mentioning that with such an indexation the pension of Pillar I and II can certainly not joined up.
 2. the bonds (securities representing credit relationship) whose return is linked to the inflation. Currently there is no bond of changing indexation on the (international market) (apart from the disaster bonds, obviously not to be considered here).

So the shifting apart can be best avoided if in Pillar I indexing to inflation is applied (this is the wide international practice), in Pillar II the rule must be indexing to inflation parallel with investing to bonds indexed to inflation.



4.3.3.4. *The alternative SoS model in the literature*

This model was raised, respectively “designed” by several experts more or less independently from each other; according to news of different sources it is being raised and designed generally not in a written form. So far I have found written proposals at two sources (though the source of the idea is not necessarily these papers and they are not even referred to in this context.)

The first proposal can be found at James (edit) [1994], in the form of a short idea, not in an elaborate manner), who raised it explicitly in respect of the annuity problems of the DC system. According to him the remaining lifetime is difficult to foresee, even on the level of cohorts, this is why the providers cover their annuity exposure by provision or by life insurances. In his view the possible solution is that private pension is only used in the first period of the pension, e.g.: in the first twenty years and at a higher age the national pension system would take over. In the national pay-as-you-go systems the cost of the higher life-span is assigned implicitly to the next generation what the state can impose by its power to levy taxes.

The second drafting was explicitly made for the Hungarian circumstances in item 20 of the paper written by Barabás-Bodor-Erdő-Fehér-Hamecz-Holtzer [2006]. According to this “The annuity payment lasting from the time of retirement until the end of the life may raise questions. This was not the case until we had only one national pay-as-you-go social security system. Nowadays though we have already have private pension fund too (what is more, we may even have other recognized pension products in the future as well). There are very few countries in the world, where one can find a competitively, therefore effectively, operating annuity markets. Hungary is not one of them; probably we will not even be so lucky in the future. We will have an oligopoly, over prices, non-transparent annuity market (why would this be the only exception?)., where we will have to convert our fund savings into annuity. If this is the case why do we have to force people to buy annuities? If we think that it is not necessary, and we allow people to have access to their property in the form on bank annuity e.g. phased withdrawal, and then it is not necessary at all, that the social security system and the second pillar start the annuity payment at the same time. Consequently we have to think about a need to revisit the principle of parallel paid



annuity and instead of that to convert to subsequently paid annuity in such a manner, that the risk of a long life be covered by the social security system, what is better fit for this risk community due its size, while in respect of the fund payments the possibility would be given, within certain restrictions and incentives, for the young pensioners to have more say in the timing of their payments: the general principle remains, from the time of retirement until death, but in respect of the different subsystems it gains a different interpretation.”

4.3.4. The insurance (return-refund) annuity model

In the literature I (Banyár [2007]) have drafted this model, and the Ministry of Finance approved it as a concept of regulation. The annuity regulation approved in December 2009 partly includes this model (and partly the model of the central provider). As preliminaries the literature presented in the chapter of indexation, specifically Réti [1999] can be brought up. In Réti’s view the indexation can not be else than return based (Réti does not yet mention the problems of managing the mortality gain, neither that it can not take place on the basis of the fund), nor he mentions Stahl [2005], according to whom the indexation should happen on the basis of the combined investment and mortality performance. Though as Stahl allow the nominal decrease of the annuity too, his proposal can be rather interpreted as one belonging to the fund annuity model.

My own concept is the changing of the provider during the maturity, what is an important element of the model. The technical literature contains only a few hints about it. Réti [1999] and Augusztinovics-Gál-Máté-Matits-Simonovits-Stahl [2002] also raise that in the annuity payment phase the provider may be changed, but it is mentioned such as it was included in the law in force, the problem and consequences whereof they do not discuss. The latter paper says only that the “Some of the provisions of the act suggest that according to the intention of the legislator movement among the providers is allowed in the course of annuity payment as well. This may have an effect of “selection of the unfittest.” The idea is also raised by Stewart [2007], among other possibilities he only mentions without an elaboration in order to encourage competition.



4.3.4.1. The essence of the model, its internal alternatives

On the grounds of the above the most important element and the internal context of the model are the following:

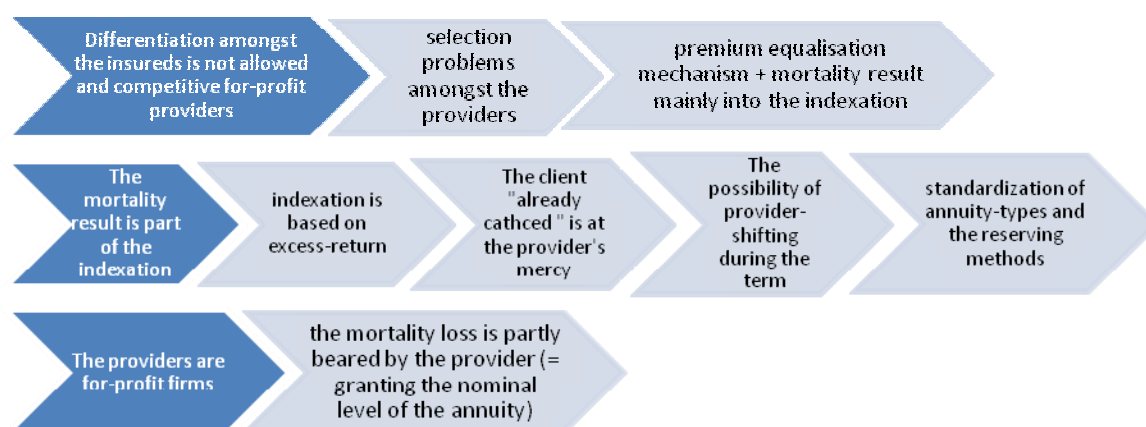
Competing or non competing providers	Clients may choose – with certain restrictions - from among several competing providers.
Provides the full annuity or only a part of it (phased withdrawal)	The providers provide the entire annuity, so among the providers the selection problem may emerge, what needs to be managed.
Managing the selection problems of the provider.	The selection problems of the provider are managed at the beginning of the term by a central minimal pool, selection problems emerging in the course of the term, e.g. selection impact hidden in the mortality loss are partly assigned to the clients via the indexation mechanism. From an other end this also means that the mortality profit also belongs to the client but only until the extent it does not endanger the nominal value of the annuity.
The provider is for profit or non for profit organization, it is the final bearer of the mortality loss.	The providers are privately owned, for-profit organizations (insurance companies, or institutions operating and regulated similarly to insurers) They may run the entire mortality risk too, but its “option fee” would be too high for the clients, so they only run a part of this risk.
Is the annuity composed of one or of two parts?	Both versions are possible but the regulator must choose, not the clients.
Indexing	Partly excess return indexing, otherwise providers’ competition would not be reasonable, partly in order to be able to manage the mortality performance, the selection problem of the provider respectively. The technical interest rate must be fixed at this indexing technique, so that self selection occurring due to different technical interest rates can be avoided. Since at this indexing technique the clients would be exposed to the provider who acquired them, because the provider does not grant (can



	not grant!) in advance a rate of indexing, therefore the competition conducted for the clients must be maintained in the annuity payment phase too, so changing of the provider must be allowed even in this phase. However this is only possible if there is a standardized provisioning, so it takes place with pre-defined technical interest rate and mortality table.
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The above context can be illustrated in the figure below:

Figure 6: Internal context of the elements of the return-refund annuity



We also receive the above model, if the starting point is that it is difficult to issue bonds indexed to some of the economic indexes (price index), or if not, so if there exist such bonds, we can not expect them to remain for long, respectively they will not be available in the desired quantity in the future. Therefore annuity of fixed service (indexed to some of the economic indexes e.g. to the inflation) is not an option, only an annuity of changeable annuity types are possible. The annuity technically operates somewhat differently in big and in small risk communities. The problem of eventually small risk community is not a question (or it does only for a short time at the starting up of the whole system) in models of central provider and of central pool, and its significance is small in models where the deferred regular annuity is combined with phased withdrawal. At this mode though the risk community is shared between the eventual many providers, so it may happen that the risk community is small, mainly after the start up of the whole annuity system, respectively after the start up of the different providers. So the mortality result may be extreme, in fact the initial



premium equalizing mechanism does not manage every problem, so these are managed by the other solutions. Therefore for the event of small risk community specific rules are required.

The above said suppose big risk community, when the mortality profit does not have extreme fluctuation. In the event of small risk community the extremely fluctuating risk community can not be made part of the indexation, but it belongs to the provider, so the mortality loss is born by the provider against its solvency capital. Therefore it is important that the provider is a for-profit organization having solvency capital

Below I elaborate on the operation in “normal” case (i.e. supposing an adequately big risk community) and in the case of small risk community.

4.3.4.2. The operation of the annuity in “normal” case

The most important technical suppositions in respect of the annuity are as follows:

- The premium is unisex, and is only dependent on age and amount of insurance, but the provisioning is differentiated according to gender and educational attainment. (As an alternative, differentiation in premium according to education should also be considered; this would greatly support the system to be blames of pervert redistribution, so that the rich are subsidized via the contribution paid by the poor. The provision is naturally differentiated according to education.)
- Indexing the annuity depends on the combined investment and mortality performance, where only a maximized part can be deducted from the gross yield to cover the property management fee. The mortality performance is separately calculated in risk groups according to gender and education, and is spread within the group.
- The unisex table used to premium calculation and the tables used for provisioning differentiated according gender and education are generated centrally. The use of central tables is mandatory for the provisioning. Each mortality table pertain the projected mortality.



- The central mortality tables based on the experience, respectively the changed expectations are regularly updated. Always the most recent tables must be used for provisioning. These tables may contain generation effect in the future, so projections are separately prepared for different age groups, and these are separately adjusted.
- The technical interest rate is centrally fixed, it is recommended to use the 0%.

The annuity is recalculated in each year, if the insured individual is still alive at the beginning of the year. The insured individuals are managed separately according to risk groups – i. e. according to gender and education – so separate mortality tables are applied for them at provisioning and on this basis the annuity of the different risk groups are newly calculated. The factors of recalculation are:

- The starting point is the previous beginning of the year amount in the individual premium provisions (“annuity account”) of the insured (annuitant).
- This decreases in the course of the year by the annuity payments during the year, and it increases by the share of provision assigned to him from the premium provision of those who died during the year⁵². This is how the computed premium provision is computed by the end of the year or by the beginning of the next year for the given annuity unit. This will be different from the actually necessary one, because in the meantime the mortality projection was modified (if it was), and the annuity unit would probably be different from the actually possible one as a result of the investment and the mortality risk.
- If the entrusted institute issued at the beginning of the year a new mortality projection, the provision need for the insured persons alive is defined on the bases of the actual annuity defined, of the age of the insured and of the new mortality table. If there is not new mortality projection, this is given based on the previous point.

⁵² Should there be no 0% technical interest rate, this would increase by the supposed yield due to the technical interest rate.



- If the so calculated provision need is higher than the provision available (what decreased compared to the one at the beginning of the previous year by the paid annuities, increased by the investment profit, and the premium of those who died is also available for those who live), then the missing provision is capped up by the insurer from the solvency capital, the and level of the annuity remains as it was in the previous year.
- If the so calculated premium need is smaller, than the provision available (presumably in the majority of the cases the situation will be this due to the projection and the 0% technical interest rate) then to the extent it is bigger, the annuity must be increase in the same rate in the given year compared to the previous year, and naturally the provision must be increased alongside.

As the result of the above method the annuity of the year increases precisely by the profit compared to the previous year if:

- Precisely as many people died in the previous year as many was expected to die based on the statistics
- The average annuity of the died is identical with the average annuity of the alive
- The mortality projection did not change compared to the one of the previous year

The annuity increases higher than the profit, if the number of died people exceeds the expected number, respectively if the average of their annuity is higher than the average of the total annuitant population, or if the mortality projections is so modified, that the remaining lifetime decreases and vice versa if a change of opposite sign takes place. Naturally in a year these factors may not necessarily move to the same direction, and we get the final annuity increase as the resultant of all these. If the factors move to identical direction (e.g. the profit was low, the actual mortality decreased – especially among those of bigger annuity – and the projected mortality decreased too, it may happen, that the annuity would decrease compared t the one of the previous year – had there be no solvency capital.



4.3.4.3. Changes of the operation in the case of small risk community

Every differentiated sub-group of the risk community (according to gender and education) must achieve a certain minimal number of members at a provider or of a part of the providers decide to unify their stocks – so they organize a pool – this is applicable for the entire pool.

If all the insured individuals of a provider or a pool altogether does not attain the minimum number of members in a year (what characteristically happens at the start up of an annuity system respectively later at the start up of a new provider) then in that given year the total mortality performance belongs to the provider, i.e. the loss will not be settled against the provision of the clients, and the profit is not distributed among them. If due to this eventually deficit emerges in the provision, it will have to be covered from the solvency capital of the provider. The provision of the clients belonging to different risk groups must be calculated in such cases by using the adequate differentiated mortality table too.

If all the insured individuals of a provider (or a pool) are altogether bigger than the minimum, but it has such a risk group, whose number is smaller than the necessary minimum, the differentiated sub-group of the smallest number must be temporarily reduced with the second (third, etc.) subgroup of smallest number until they reach the minimal number, then that (respectively those) must be managed according to the normal conditions, though the reduced subgroups must be managed in a consolidated manner what means that the mortality profit of the consolidated group is jointly managed – while we further on use the separate mortality tables for the provisioning.

The above examination must be conducted every year, and on these grounds the consolidated group may cease to exist, or there may be a need for consolidation in the next year.

4.3.4.4. Transitions, timing

The different providers' timely coordinated activity in respect of the annuities is important because of the competition prevailing in the phase of annuity payment, so a precise timing of activities has to be made. It follows as well, that the timeline of thinking should be the calendar year as it is uniform for everyone, not like the insurance year.



A possible timing of the annuity related activities is the following. The significant changes take place in the first quarter of the calendar year:

- In the first two month of the year the insured persons must report their intention to transit to an other provider; from the increase of the annuity it takes place.
- At the beginning of the third month the control of the risk groups takes place, once the transitions are known, and if necessary the consolidate risk group is defined.
- We understand that the dead insured person is who died in the previous calendar year, the total of his actual denominated account is distributed, from which the total of the actually paid annuities is deducted and we add the combined mortality and return profit distributed in the previous year.
- The actual number of the dead (and of course their name), respectively the return of the previous calendar year is defined in March.
- By the end of February the most up-to-date mortality projection must be prepared (if we agree in annual adjustment. However the adjustment of the mortality projection may be rarer, but this does not change the method, it must be completed by this time.)
- At the end of the first quarter (in March) we define the combined investment and mortality profit and the rate of the buffer – with regard to those who transit (both from a provider as well as to a provider) -
- From the beginning of the second quarter (from April) the annuities may be actually increased (if at all).

An other manageable factor is that the new entrants, and those who died during the year are not included in the above computation, they will be taken into account – from this aspect – only in the coming year. Naturally the provider pays immediately annuity to the new entrants, and the annuity payment is immediately terminated for those who died.

4.3.4.5. The premium settlement mechanism

A uniform measure is required for the premium settlement mechanism; therefore the market needs a centrally prepared unisex, projected mortality table. This



is not a problem, since it is anyhow available at this model (to allow changing of the provider during the maturity).

When a client buys annuity from a provider he pays a net premium calculated on the basis of a centrally determined unisex mortality table plus he pays for the costs. To meet its mandatory provisioning requirement the provider immediately calculates the risk by using the differentiated mortality table adequate to the gender and education of the client. There is a difference between the actually collected net premium based on the unisex mortality table and the so calculated necessary provision that may be even bigger than the unisex premium (e.g.: in the case of women with diploma) or may be lower (e.g.: in the case of man of elementary education). The difference must be transferred to a central organization, a clearing house operating the settlement mechanism; respectively the providers receive transfers from there. There may be an option in respect of the mode of operation; there are arguments for and against making the settlement by case or by accumulated by periods.

As I have already mentioned the problem may be in the case of this solution, that losses and profits generated during a period of time at different providers would not necessarily offset each other, since on the one hand the unisex table is not certainly accurate, on the other hand clients representing identical risk for the provider may not conclude annuity contracts all over the annuity market at every moment. If the central mortality projections were correct, the balance of profit and loss was zero on long term, but it is almost certain that on short term this is not the case.

I would propose to connect the final balance generated by the premium settlement mechanism with the preparation of central mortality projections and one of the guarantees for the adequacy of the projections is that this must be balanced on long term. The temporary deficit is covered by the state but it is repaid from the transitory surplus, though the surplus must also be looked at as transitory, so money must not be withdrawn from the annuitant.

The neutral balance of the premium settlement mechanism though does not make necessary that the projection of the life expectancy be correct, therefore this rule is not the entire guarantee for the mortality projection, only of the correctness of the unisex table. What's more the system may have a neutral balance also by



manipulating the life expectancy; therefore further rules are required to test the adequacy of the projections.

4.3.5. The model of the central pool

The model was first proposed by György Németh (Németh [2006]), the central institute of the pool is the so called “government actuarial” who would redistribute the mortality result among the providers (in his proposal these are clearly the funds of our days). Németh finds, that there is no justification for differences in the premium offered by the funds if there is no possibility for differentiation, moreover different premium would lead to operational distortions, what can be prevented by having a single provider (in reality by a central pool – JB). In his view this does not mean a single organization, but a single set of rules – contrary to Stahl’s proposal promoting the idea of a central provider.

4.3.5.1. The essence and internal alternates of the model

Based on the above the most important items and the internal context of the model is the following:

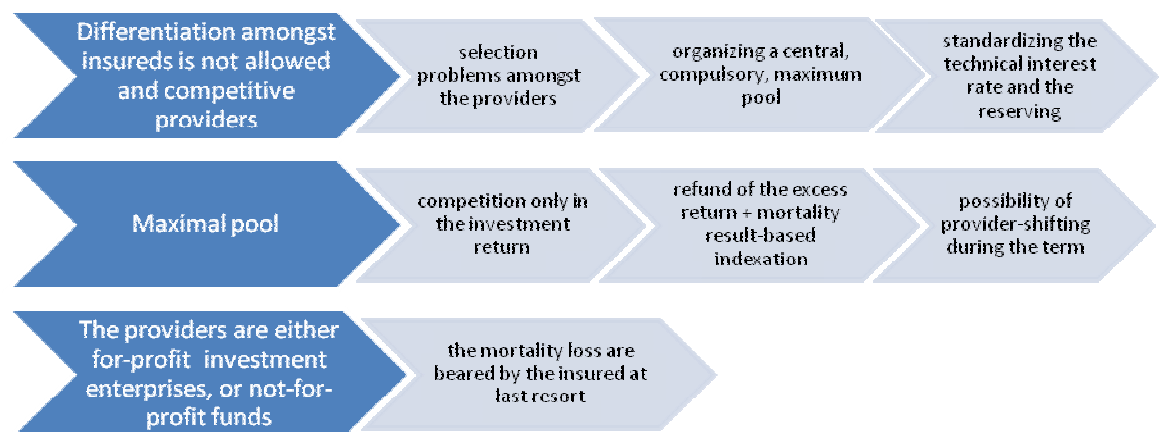
Competing or non competing providers	Several competing providers provide the service among whom the clients may choose – with certain restrictions.
Provides the full annuity or only the first part of it	The different providers provide the full annuity, so the problem of selection emerges among the providers, what must be managed. However the wording that the providers “provide the annuity” is somewhat inaccurate in the case of this model in the light of the followings.
Managing the selection problems of the provider. (The final bearer of the mortality loss)	The selection problem of the provider is managed by the help of a maximum pool, so the mortality result is distributed by the help of a maximum pool. Inevitably this means that the technical interest rate and the provisioning is fixed, centrally determined. The mortality result so remaining in the entire market is basically of the insured individual.
The provider is a for profit or a non-for-profit	The providers may theoretically be non-for-profit companies (like for example the actual Hungarian funds), as well as for profit



organization	companies (basically investment ventures – therefore they may not bear the mortality loss in the end, since they are institutions of not insurance type).
The annuity is composed of one or of two parts	Both solutions may be possible depending on the decision of the regulator, but only one of them may be in the market.
Indexing	Anyhow there is an excess return refund, as this is the only solution where competition in investment between the providers is reasonable. Consequently the clients may change provider even when the annuity payment started, what is not excessively difficult to organize due to the centralization of the service made according to a certain aspect. Indexing the individual clients depends on the total of the yield achieved by their provider and on the actual mortality result – that may even be negative, though this possibly rarely happens.

The above context may be presented on the below figure:

Figure 6: Internal context of items in the central pool model



Obviously the different providers must precisely determine the amount of money they have to invest (the individual premium provision of their insured individuals), so it is reasonable to restrict the option of changing the provider to a certain period of the year, possibly for the very period when the pool determines the mortality result of the previous year, and by this the index.. In addition to the above, this is the occasion, when the clients may face the result of their investor. In this model



the full annuity service may be centralized, even it is reasonable including the client registration. In order to do so the different providers do not even have to know who their clients are. If they don't know, the intermediaries may be eliminated from the system, by this cost may be saved, though the role of the intermediaries is not quite clear, since many clients are made aware of the possibility to leave the under performing providers by the intermediaries.

4.3.5.2. What service does the provider give, or should it be for profit or rather non-for profit organization?

The internal alternate of the model is that the provider is a non-for profit organization (in Hungary these are the actual funds), or for profit organization; it is worthwhile to review the arguments for one or for the other solution.

The argument in favor of the funds is that the existing institution, who though are inadequate by themselves - in the lack of solvency capital – to provide adequate life annuity. By this solution the annuity provision may be organized on the basis of the actual funds. In the case of the central pool the administrative work and the membership registration is reasonable to centralize partly because this helps the calculation of the mortality result, partly because if a central institution is established anyhow it is reasonable to organize the respective activity there. If we compare this with the fact that the majority of the funds do not conduct investment activity, but entrust investment companies with that the fund, regarding its activity is “emptied”, so it is worthwhile to think about “disconnecting” the funds from the annuity system in the central pool model, and to entrust the investment companies directly with the investment activity directly.

So the idea that the competing providers should be for-profit institutions is strongly promoted.

In theory two different institutions may “apply” for this, the insurers and the investment enterprises. Though in the case of insurers the central pool means a very strong diminishing of the risk, so this is not a tailor made solution for those institutions. Therefore in the case of insurers rather the minimum pool can be proposed instead of this, what paves the way for an other model, the above insurer annuity model. So the investment enterprises remain. Naturally these can not even



bear the mortality loss (respectively it can be imagined on long term, that they would invest into 'longevity bonds', if they do yes, they can!), so contrary to the other models – finally the mortality loss is born by the insured even in the case of the for-profit companies; the mortality risk though is almost totally purified from the haphazard fluctuations due to the possible biggest risk community, so this only means the longevity risk.

4.3.6. The model of the fund annuity

The most important elements and their internal context are as follows in the model:

Competing or non-competing providers	Several, competing providers give service, from among whom – with certain restrictions – the clients may choose.
Provides the full annuity or only the first part of it (phased withdrawal)	The competing providers provide only the first part of the annuity, in the framework of phased withdrawal (so not as annuity). This partly manages the problems of selection resulting from the prohibition of differentiation.
The annuity is composed of one or of two parts?	The consequence of the above is that the annuity is composed of two parts, where the first unit is the phased withdrawal what allows the insured individuals to postpone the start of the annuity, and to leave the majority as a potential heritage (often referred to as "the client has discretion over his money"). However the insured individual must immediately buy the deferred part of the annuity.
Managing the selection problem of the provider	The remaining selection problems, what are a priori smaller, as if the provider gave immediately starting annuity, and what is born by the deferred annuity unit, is delegated by the provider to a central provider – by transferring the deferred annuity, so the provider is discharged from that.
The provider is a for-profit or a non-for profit organization (final	The provider is a non-for profit company (among Hungarian circumstances) it is a fund as the name indicates. Though as it can not provide life annuity under no circumstances, does not bear

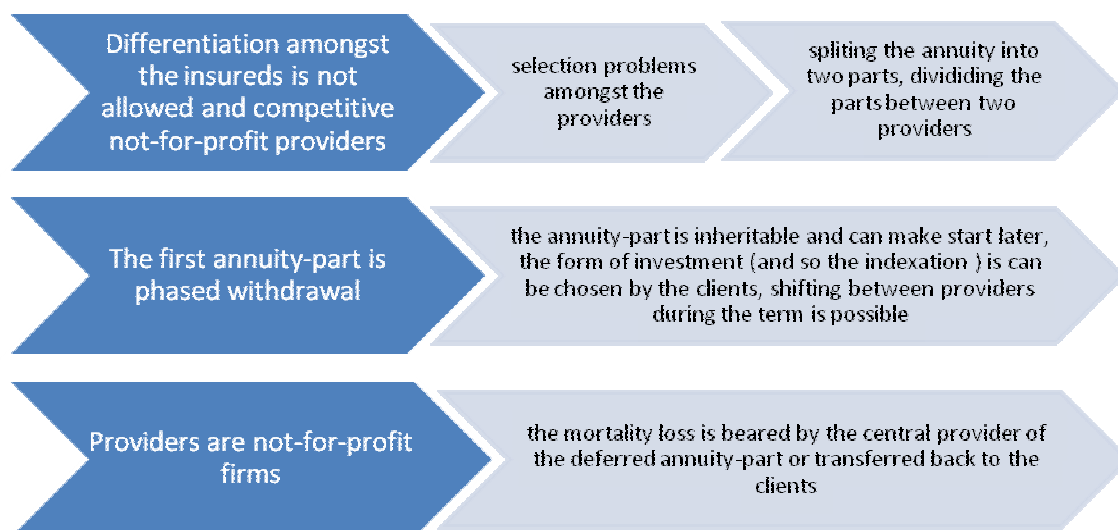


bearer of the mortality loss)	the mortality loss either, that may only emerge at the central provider. The provider may manage mortality risk as it is managed in the model of central provider.
Indexing	<p>Theoretically indexing may be solved by both indexing technique, and arguments may be raised for and against both solutions. In favor of indexing to inflation the argument is that the two annuity units are relatively seamlessly adjusted to each other, though it is fairly difficult to organize the changing of the provider during the maturity, however without this in the case of the other indexing technique the “already acquired clients” are exposed to the provider being in a monopolistic situation. An argument against is that organizing the service by the competing providers, this may only be the case if the competing providers are taken as granted.</p> <p>In the case of indexing on the bases of excess return it is justified to assure the clients the possibility for both changing the provider and for choosing the investment portfolio.</p>

The above context can be presented in the below figure:



Figure 7: The internal context of the items in the Fund annuity model



I have already written everything what is important about the details of the model⁵³, so I do not intend to make a repetition. It is important to note though that as the fund may not bear mortality risk – in the lack of solvency capital - the client may not choose at all (temporary) life annuity during the deferred period, he may live up his savings (or may leave it as legacy) only and exclusively in the framework of scheduled money withdrawal. In the case of fund annuity providing full annuity for own responsibility is excluded because – in the lack of solvency capital – there is no penalty on bottom pricing, so on short term it is worthy to attract clients by irresponsible promises, on long term the providers may not be able to give service at most. The problem of possible, unpunished, bottom priced life insurances therefore might occur only at this model; therefore the possibility must be excluded due to this consideration!

⁵³ Despite that for long the Hungarian pension profession took as granted that the private pension annuities would be provided by funds, the fund annuity model is not developed in the technical literature. This is related to the fact that the rules on private pension annuities in general were not designed in Hungary until most recently. For long there was an illusory thinking about the annuities provided by the fund, it was assumed that it is not different in any way from the annuity provided by the insurers - at most the legislators set certain restrictive conditions as for example the Swiss indexation, but these were not deemed problematic. The exception was János Stahl, who found both unisex annuity and Swiss indexation problematic; therefore Stahl in [2005] proposed the combined investment and mortality result based indexation, allowing that the annuity nominally diminishes, what is the essence of the fund annuity model. However he also proposed to have a central provider too, so he did not systematically elaborate the fund annuity model.



4.4. MODEL ELEMENTS APPLICABLE RELATIVELY VARIABLY TO SEVERAL MODELS

Above I have elaborated on those elements of the annuity system that are not optionally variable, the selection whereof supposes the selection of other elements, respectively excludes the selection of other elements. Nevertheless there are relatively neutral solutions too, which may be inserted – under certain conditions – to any annuity model.

4.4.1. Homogenization, or the maximum annuity

Homogenization annuities (obviously partially) according to the magnitude may be raised in respect of every annuity model. This homogenization diminishes the risk of the generation of large mortality loss, because though the mortality is properly projected but the bigger annuitants systematically live longer than the smaller ones. The emergence of the problem may also be deducted from the prohibition of differentiation, since the magnitude of the annuity presumable correlates well with such factors that probably can not be taken into account at the premium, such as educational attainment or health condition. Homogenization increases the precision of the calculation at each annuity model, but this is emphatically true for models using immediately starting annuities. Consequently implementation of homogenization is highly recommended, e.g. the maximization of the possible annuities.

The issue of homogenization is raised not only from this perspective, but also as a matter of free disposal of the pension assets of the insured persons. The question is whether it is worthy to oblige the insured individuals being above a certain pension level to convert their savings to annuity. I think that the answer is no, as the objective of mandating is that everybody attains a certain level of pension, so that people are safeguarded against an eventual short sighted economy. People can be left to assure a level beyond that according to their wish, respectively short sighted economy does not raise social problems beyond a certain level of pension. Luckily raising the issue of homogenization in two different ways converges to the same solution.

Naturally the method of setting the annuity maximum is raised. There are two options:



1. Independently for the annuity of Pillar II. In the alternate SoS model this is the entire pension annuity at the same time.
2. for the annuities of Pillar I. and II. by consolidation, e.g. the annuity of Pillar II. may be maximum so much as to complement of the annuity of Pillar I to a certain level. Ad absurdum this may be even 0, if the pension from Pillar I. also attains this minimum.

Option 1. is easier to managed within Pillar II. as in this way there is no need for data derived from Pillar I. and it probably leads to more homogeneous annuities too than option 2. However despite of this the problem of inhomogeneous is better managed by option 2, because probably there is a tendency that people having bigger savings have longer life and also get higher pension from Pillar 1 too, so their annuity from Pillar II. would be below the average. Naturally when selecting option 2. one must take into account that as a result the annuity from Pillar II may be zero, but may not be below a minimum level (see at the elaboration of “minimal annuity”).

Whatever will be the decision of the legislator; the insured person may withdraw as a lump sum the portion beyond the level of assets necessary for the annuity maximum, and may use it according to his discretion. (An other possibility it to buy non-mandatory annuity in the free market, but may leave it as legacy or may invest it too.) In the annuity models starting with phased withdrawals this issue raised so, that the first withdrawn amount may be a bigger amount too, what may be defined according to these principles.

There is no problem to insert such a rule into every annuity model.

Naturally we may raise the question why the clients are obliged to accumulate money at all in a mandatory pension system, if they are not obliged to use this money as annuity in this framework. There are experts who say that if there is an upper limit in respect of the current income for payment into Pillar II (e.g. in Pillar II in Hungary), this defines the annuity maximum as well too, so this aspect of the matter is not to be dealt with. Though this is partly true but in the case of maximum determined in the income the eventual annuities are spread among wide bands, so they do not give a precise maximum because:



1. the income may change, fluctuate respectively: it either exceed the maximum, or there is no income (neither pension contribution) for a period of time at all during the life of an individual (it may frequently happen with women due to child raising when they drop out of work)
2. the magnitude of the annuity also depends on the given age of retirement a. Projected to identical capital it is significantly higher at a higher age of retirement than in the case of early retirement.
3. the remaining life expectancy also changes in the course of time, consequently the amount of capital needed for identical pension minimum changes to an unforeseeable extent.

In addition such a minimum pension must be indexed too, what may not necessarily be parallel with the return of the already accumulated pension assets. Due to all these it is impossible to precisely calculate in advance whether the later annuity exceeds a certain maximum or not, therefore a certain amount of over slipping certainly takes place at a part of the insured individuals even if there is a well determined annuity maximum. The two rational considerations that:

1. it should be assured to the possible extent that the future pension gets as near as possible to this maximum
2. it should be avoided that the insured individuals are needlessly obliged to accumulate more than it is necessary for buying the maximum annuity

may lay the foundation for the future reform of Pillars II (e.g. the Hungarian Pillar II.) In a system with regard to the above said the pension contribution to Pillar II does not have a maximum tied to income, though there is a maximum (calculated with a margin set due to the above uncertainty factors) pension capital to be accumulated, what is counted down from the maximum annuity level. If the private pension savings attains this level, payment to Pillar II terminates. By this we can avoid that an insured person with promising carrier will have a low pension only because he had many times high income during his life but at those times he did not pay contribution at an adequate level, and in other phases of his life his income was low, or he had no income at all.



Naturally an argument against this solution is that pension contribution must primarily be paid at a young age, when the money is theoretically needed for other things too, but there are arguments against this (e.g. the consumption level of the young ones is set so at a low level, or the lower net income poses a bigger effort, what is easier to make at an early age than at an older age, etc.).

We may logically rise that if the issue of mandatory maximum annuity is raised similarly the issue of minimum annuity may be raised. I elaborate on this problem in the coming chapter in relation to the regulation.

4.4.2. The two lives annuity, as a possible element of any model

So far the default was as each model that in order to eliminate self selection as the result of possibility of choice among annuities the annuitants may not select the type of annuity, it is defined for them by the model. If there is only one kind of annuity in a model, this condition is met. Though it is also met, if there are different types of annuities, but they are clearly assigned to the different client groups. In practice this means that if singles are given single life annuity and two lives annuity is given for couples⁵⁴. Therefore the two lives annuity may theoretically be inserted into any annuity model too.

The different models above included two types of annuities:

1. immediately starting, single life, simple (without guarantee time , life long) annuity
2. deferred, single life annuity, possibility for phased withdrawal during the deferral (from the individual provision separated from the premium of the deferred annuity)

I think that the two lives annuities must adjust to the single life annuity type of the model, so the immediately starting, simple, two lives annuity, or the deferred two lives life annuity.

The exact structure of the two lives life annuity should be precisely designed. Above I proposed a possibility, but than can be further fine tuned by taking into account the specific different life conditions (big difference in age between the souses,

⁵⁴ Or to people living in any documented life community what is not easy to change, and what is precisely defined by the law..



big difference at the beginning of the pension, one of the spouses works the other does not, etc.), and on this basis the rules of the annuity should be refined.

Similarly divorce and marriage has an impact on the type of the annuity, what must be taken into account and the rules must be refined accordingly, with specific regard to the possible irregular situations (new marriage of widows, divorce before the retirement, or after the retirement of one of the spouses, etc.).

The two lives annuity may solve the problem of widow's pension in Pillar II. Respectively this statement must be somewhat refined. Two different benefits are generally means by widow's pension, there is an immediate pension (right after the death of the spouse):

1. temporary annuity, in the event of death in active age
2. life long annuity, in the event of death at the age after retirement.

The two lives annuity may solve the problem of widow's annuity of type 2. Type 1 is managed by Pillar II. by the inheritance of the pension accounts in theory, but this is not necessarily the best management of the problem, because there may be many different amounts on the pension account of the spouse – compared to the lost income of the spouse - , depending on at which phase of the lifecycle he happened to die. In addition to this the heir may often be freely assigned (e.g. in currently in Hungary), therefore this amount would not necessarily get to the widow. Therefore instead of this the implementation of the following two rules should be considered:

1. all the temporary widow's pension due in the event of death taking place at an active age in reality is a life insurance, this should be explicitly inserted into the system, and its premium should be separated from the pension account
2. the pension account is inheritable, but the heir is clearly the spouse if they lived in marriage, and the accumulated pension assets get onto the pension account of the spouse, and later that will increase the later old age pension.

4.4.3. Implementation of the individual provision (buffer)

The individual provision or buffer is to smooth the mortality fluctuations, respectively to bear the final mortality risk (naturally up to the amount of the buffer)). This solution should be considered in each of those models, where the final bearer of



the mortality loss is the insured individual – so primarily in the central annuity model, in the annuity model of the fund, in the model of central pool. It should also be considered at cases where the for-profit provider gives immediately starting annuity (so in the insurance life annuity model), so in cases when

- a. on the one hand the fluctuation of the mortality loss on the one hand has an impact on the bigger portion of the maturity, so altogether the accumulated loss may be bigger than in the models using deferred life annuity, respectively
- b. on the other hand the mortality loss is born as a main rule by the provider, therefore it may levy too high “option fee”, so it may too much diminish the paid annuity.



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